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A THEORETICAL INVESTIGATION OF GAS-BUBBLE IMPLOSIONS IN LIQUIDS

by

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

a	A constant for the Beattie-Bridgeman equation of state, (see Table 1, Equation [1.8])
$\bar{A}$	A constant for the constant pressure heat capacity equation for the ideal state (see Table 1)
A_o	A constant for the Beattie-Bridgeman equation of state (see Table 1, Equation [1.8])
b	A constant for the Beattie-Bridgeman equation of state (see Table 1, Equation [1.8])
B	A constant which characterizes the adiabatic compression of the liquid
$\bar{B}$	A constant for the constant pressure heat capacity equation for the ideal state (see Table 1)
B_o	A constant for the Beattie-Bridgeman equation of state (see Table 1, Equation [1.8])
c	A constant for the Beattie-Bridgeman equation of state (see Table 1, Equation [1.8])
c_∞	Sound speed in the undisturbed liquid
c_p	Instantaneous specific constant pressure heat capacity of the gas inside the bubble
c_p^o	Instantaneous specific constant pressure heat capacity of an ideal gas inside the bubble
c_v	Instantaneous specific constant volume heat capacity of the gas inside the bubble
c_v^o	Instantaneous specific constant volume heat capacity of an ideal gas inside the bubble
C	Instantaneous isentropic sound speed in the liquid at the cavity wall
$\bar{C}$	A constant for the constant pressure heat capacity equation for the ideal state (see Table 1)
$\bar{D}$	A constant for the constant pressure heat capacity equation for the ideal state, (see Table 1)
H	Instantaneous specific enthalpy of the liquid at the cavity wall
n	A constant which characterizes the adiabatic compression of the liquid
p	General liquid pressure
p_∞	Pressure in the undisturbed liquid; ambient pressure
P	Instantaneous pressure of the gas inside the sphere

P_o	Initial pressure of the gas inside the sphere
R	Instantaneous radius of the imploding sphere
$\bar{R}$	The gas constant
R_o	Initial radius of the imploding sphere
s	Specific entropy of the gas inside the bubble
t	Time
T	Instantaneous temperature of the gas inside the bubble
u	Instantaneous specific internal energy of the gas inside the bubble
U	Instantaneous velocity of the bubble wall
v	Instantaneous specific volume of the gas inside the bubble
v_o	Initial specific volume of the gas inside the bubble
γ	Ratio of specific heats, $\frac{c_p}{c_v}$, for the gas
ρ	Liquid density
ρ_∞	Density of the undisturbed liquid

ABSTRACT

Two methods are presented for calculating the instantaneous pressure, velocity, acceleration, and radius associated with the collapse of a spherical gas-filled cavity in an infinite compressible liquid. One is based on the ideal gas law, the other is based on the Beattie-Bridgeman equation of state for the gas inside the cavity. In most cases the latter assumption must be restricted to relatively mild implosions. The good agreement between the two methods serves to verify their validity.

Included are listings of the two Fortran IV computer programs used to obtain numerical results of the analyses based on the ideal and Beattie-Bridgeman gas models. The influence of several different gases, initial internal gas pressures, and liquids on the collapse is studied. On the basis of explanations of the resulting behavior, new methods of producing similar behavior are discussed.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

Due to their low density and high compressive strength under hydrostatic loading, spherical glass shells have a promising application in the design of deep submergence vehicles.[1,2] However, due to the nature of fracture of glass shells, the underwater environment created by their failure (implosion) at great depths is very similar to that of an underwater explosion. Consequently, the effects of an imploding glass sphere on neighboring objects, especially other hollow glass spheres, must be given careful consideration if hollow glass spheres are ever to be at all suitable for all-depth vehicles. Because the sequence of events associated with the failure of a single glass sphere in a free liquid field can be very closely represented by a gas bubble implosion, the latter is of primary interest. In this paper, an extensive theoretical investigation of the free-field implosion of a spherical gas-filled cavity in liquid is presented. This paper is intended to supplement a previous paper on the subject.[3]

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[1]References are listed on page 75.

THEORY

EQUATIONS GOVERNING BUBBLE WALL MOTION

Gilmore[4] has derived an ordinary, second-order, nonlinear differential equation which relates the instantaneous pressure of the gas inside a collapsing (nonmigrating) spherical cavity in an infinite compressible liquid to the instantaneous radius of the cavity. Briefly, the equation is obtained by employing the Kirkwood-Bethe hypotehsis[5] and basic fluid flow relations to solve the spherical wave equation. The details of the derivation can be found in Gilmore's report.

If R represents the instantaneous radius of the cavity, P the instantaneous pressure of its boundary, and t, time, Gilmore's equation is

$$R \frac{d^2R}{dt^2}\left(1 - \frac{1}{C}\frac{dR}{dt}\right) + \frac{3}{2}\left(\frac{dR}{dt}\right)^2\left(1 - \frac{1}{3C}\frac{dR}{dt}\right)$$

$$= H\left(1 + \frac{1}{C}\frac{dR}{dt}\right) + \frac{R}{C}\frac{dH}{dt}\left(1 - \frac{1}{C}\frac{dR}{dt}\right) \qquad [1.1]$$

C is the local instantaneous isentropic sound speed in the liquid at the cavity wall,

$$C = c_\infty \left(\frac{P+B}{P_\infty+B}\right)^{(n-1)/2n} \qquad [1.2]$$

and H is the local instantaneous specific enthalpy of the liquid at the cavity wall,

$$H = \frac{n(p_\infty+B)}{(n-1)\rho_\infty}\left[\left(\frac{p+B}{p_\infty+B}\right)^{(n-1)/n} - 1\right] \qquad [1.3]$$

c_∞, p_∞, and ρ_∞ denoted the sound speed, pressure, and density, respectively, associated with the liquid when it is in the undisturbed state. B and n are constants (for water B $\simeq$ 3,000 atmospheres, n $\simeq$ 7) in the formula:

$$\frac{p+B}{p_\infty+B} = \left(\frac{\rho}{\rho_\infty}\right)^n \qquad [1.4]$$

which closely fits the isentropic compression curve for the pressure p and density ρ of many liquids. (Except for very large or very small (cavitation) bubbles, isentropic hypothesis for the liquid is justified, because the event occurs so quickly that there is little time for appreciable heat exchange to take place.)

For $t > 0$, the pressure P of the liquid at the cavity wall will be the same as the pressure of the gas inside the cavity provided the pressure of the gas is uniform throughout the cavity and the effects of surface tension and viscosity of the liquid are negligible.

The three Equations [1.1] to [1.3] establish one differential relationship between R(t) and P(t). But this alone is not sufficient to determine the behavior of the bubble, so another relationship is sought. By assuming that the gas inside the bubble obeys some thermodynamic equation of state and that the change in specific entropy across the bubble wall is negligible throughout the collapse (for the same reason that an isentropic process in the liquid was assumed), it is possible to find two independent relationships between the pressure, temperature, and specific volume (proportional to $\frac{4}{3}\pi R^3$) of the gas. These two relationships, taken with Equations [1.1] to [1.3] and appropriate initial conditions from a determinate system of equations which can be solved numerically for the instantaneous temperature, pressure, and specific volume associated with the gas inside the cavity.

From a computational standpoint, the ideal gas law is an advantageous choice for an equation of state. When an ideal gas behaves isentropically, the two independent thermodynamic relations, i.e., the equation of state and the equation which describes a zero entropy change, can be readily combined to eliminate temperature from the calculations, i.e.,

$$Pv^{\gamma} = \text{constant} = P_o v_o^{\gamma} \qquad [1.5]$$

where γ is the specific heat ratio, empirically determined for most gases, and the subscript o refers to some initial state. Since the specific volume varies as the cube of the radius,

$$\frac{v_o}{v} = \left(\frac{R_o}{R}\right)^3 \qquad [1.6]$$

P can be determined directly as a function of R by eliminating $\left(\frac{v_o}{v}\right)$ between Equations [1.5] and [1.6], yielding

$$P = P_o \left(\frac{R_o}{R}\right)^{3\gamma} \qquad [1.7]$$

Equation [1.7] and Equations [1.1] to [1.3] constitute a set of simultaneous equations whose solution can be obtained numerically. Such a solution has in fact been obtained for an air bubble in water,[5,6,7] and is easily extended to other gases and liquids. Results of such an extension are presented later in this paper.

A thermodynamic equation of state which is more accurate than the ideal gas low for gases at high pressures, but not quite as simple to apply in most cases is the Beattie-Bridgeman equation of state. In the case of nitrogen, for example, the Beattie-Bridgeman equation is accurate in the pressure range from one atmosphere to 15,000 atmospheres,[8] except near the critical point.

In this paper the behavior of a spherical gas-filled bubble in a compressible liquid will be determined numerically by assuming that the bubble wall obeys Equations [1.1] to [1.3], that the gas inside the bubble obeys the Beattie-Bridgeman equation of state, and that the expansion and compression process of the gas is isentropic (reversible and adiabatic).

Letting v, P, and T represent specific volume, pressure, and temperature, respectively, the Beattie-Bridgeman equation is

$$Pv^2 = \bar{R}T\left[v + B_o\left(1 - \frac{b}{v}\right)\right]\left(1 - \frac{c}{vT^3}\right) - A_o\left(1 - \frac{a}{v}\right) \qquad [1.8]$$

where $\bar{R}$ is the gas constant (= 0.73032 atm ft^3/mole^{o}R). The constants A_o, B_o, a, b, and c, have been empirically determined for a large number of gases.[9,10] Values for some of these gases are listed in Table 1. Many gases can be uniquely specified by these five constants so that the Beattie-Bridgeman equation represents a family of equations.

If, during the collapse, no heat is exchanged between the gas and the liquid, then each undergoes an isentropic process which determines a mathematical relationship between v, P, and T. This relationship is independent of the equation of state. Thermodynamically speaking, the change in the specific entropy of the gas is zero,

$$ds = o \qquad [1.9]$$

Since the process is assumed to be reversible, s is a function of any two of the state variables P, v, and T, and ds is an exact differential. Equation [1.9] will now be used to develop a differential expression involving P, v, and T. This expression will be independent of Gilmore's equation and the Beattie-Bridgeman equation of state.

Let s be a function of P and T, i.e.,

$s = s(P,T)$

Then, by the chain rule of differentiation, ds for an isentropic process is

$$ds = \left(\frac{\partial s}{\partial P}\right)_T dP + \left(\frac{\partial s}{\partial T}\right)_p dT = 0 \qquad [1.10]$$

Multiplying through by T and rearranging yields,

$$T\left(\frac{\partial s}{\partial T}\right)_p dT = -T\left(\frac{\partial s}{\partial P}\right)_T dP \qquad [1.11]$$

But since

$$T\left(\frac{\partial s}{\partial T}\right)_p = c_p \qquad [1.12]$$

by definition, Equation [1.11] becomes

$$c_p\, dT = -T\left(\frac{\partial s}{\partial P}\right)_T dP \qquad [1.13]$$

Finally, by employing the Maxwell relation

$$\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \qquad [1.14]$$

Equation [1.13] can be written as

$$c_p\, dT = T\left(\frac{\partial v}{\partial T}\right)_p dP \qquad [1.15]$$

The quantity $\left(\frac{\partial v}{\partial T}\right)_p$ can be calculated from the Beattie-Bridgeman Equation [1.8].

The specific constant pressure heat capacity c_p can be found explicitly as a function of P, v, and T (or any two of these, since one can be eliminated by means of the equation of state). It happens that such an expression for c_p for a gas which obeys the Beattie-Bridgeman equation appears in "Chemical Process Principles."[11] The details of the derivation are given in Appendix A.

$$c_p = c^0_p + \frac{6c\bar{R}}{T^3}\left(\frac{1}{v} + \frac{B_o}{2v^2} - \frac{bB_o}{3v^3}\right) - \bar{R}$$

$$+ T\left(v + B_o - \frac{B_o b}{v}\right)\left(\frac{\bar{R}}{v^2} + \frac{2c\bar{R}}{v^3T^3}\right)\left(\frac{\alpha' v^4 + \beta' v^3 + \gamma' v^2 + \delta' v}{\alpha v^3 + 2\beta v^2 + 3\gamma v + 4\delta}\right) \quad [1.16]$$

where

$$\alpha = \bar{R}T \quad [1.16a]$$

$$\beta = -A_o + B_o\bar{R}T - \frac{c\bar{R}}{T^2} \quad [1.16b]$$

$$\gamma = aA_o - bB_o\bar{R}T - \frac{cB_o\bar{R}}{T^2} \quad [1.16c]$$

$$\delta = \frac{bcB_o R}{T^2} \quad [1.16d]$$

$$\alpha' = \bar{R} \quad [1.16e]$$

$$\beta' = B_o\bar{R} + \frac{2c\bar{R}}{T^3} \quad [1.16f]$$

$$\gamma' = -bB_o\bar{R} + \frac{2cB_o\bar{R}}{T^3} \quad [1.16g]$$

$$\delta' = -\frac{2bcB_o\bar{R}}{T^3} \quad [1.16h]$$

and c_p^o is the specific constant pressure heat capacity of the ideal state, i.e.,

$$c_p^o = \bar{A} + \bar{B}T + \bar{C}T^2 + \bar{D}T^3 \quad [1.16i]$$

A, B, C, and D are empirically determined constants[12] which differ for different gases. Values of A, B, C, and D are given in Table 1 for some gases. Equation [1.15] with [1.16] substituted constrains P, v, and T to lie on a surface which characterizes the adiabatic behavior of the gas.

The Equations [1.16], [1.15], [1.8], [1.1] to [1.3], and the relationship [1.6] between v and R taken simultaneously form a determinate system which can be solved numerically. The equations are rewritten here for the reader's convenience.

$$R \frac{d^2R}{dt^2}\left(1 - \frac{1}{C}\frac{dR}{dt}\right) + \frac{3}{2}\left(\frac{dR}{dt}\right)^2\left(1 - \frac{1}{3C}\frac{dR}{dt}\right)$$

$$= H\left(1 + \frac{1}{C}\frac{dR}{ct}\right) + \frac{R}{C}\frac{dH}{dt}\left(1 - \frac{1}{C}\frac{dR}{dt}\right) \qquad [1.1]$$

$$C = c_\infty\left(\frac{P+B}{p_\infty+B}\right)^{\frac{n-1}{2n}} \qquad [1.2]$$

$$H = \frac{n(p_\infty+B)}{(n-1)\rho_\infty}\left[\left(\frac{P+B}{p_\infty+B}\right)^{\frac{n-1}{n}} - 1\right] \qquad [1.3]$$

$$\frac{v_o}{v} = \left(\frac{R_o}{R}\right)^3 \qquad [1.6]$$

$$Pv^2 = \bar{R}T\left[v + B_o\left(1 - \frac{b}{v}\right)\right]\left(1 - \frac{c}{vT^3}\right) - A_o\left(1 - \frac{a}{v}\right) \qquad [1.8]$$

$$c_p dT = T\left(\frac{\partial v}{\partial T}\right)_p dP \qquad [1.15]$$

$$c_p = c_p^o + \frac{6c\bar{R}}{T^3}\left(\frac{1}{v} + \frac{B_o}{2v^2} - \frac{bB_o}{3v^3}\right) - \bar{R} \qquad [1.16]$$

$$+ T\left(v + B_o - \left(\frac{B_o b}{v}\right)\right)\left(\frac{\bar{R}}{v^2} + \frac{2c\bar{R}}{v^3T^3}\right)\left(\frac{\alpha' v^4 + \beta' v^3 + \gamma' v^2 + \delta' v}{\alpha v^3 + 2\beta v^2 + 3\gamma v + 4\delta}\right)$$

$$\alpha = \bar{R}T \qquad [1.16a]$$

$$\beta = -A_o + B_o\bar{R}T - \frac{c\bar{R}}{T^2} \qquad [1.16b]$$

$$\gamma = aA_o - bB_o\bar{R}T - \frac{cB_o\bar{R}}{T^2} \qquad [1.16c]$$

$$\delta = \frac{bcB_o\bar{R}}{T^2} \qquad [1.16d]$$

$$\alpha' = \bar{R} \qquad [1.16e]$$

$$\beta' = B_o\bar{R} + \frac{2c\bar{R}}{T^3} \qquad [1.16f]$$

$$\gamma' = -bB_o\bar{R} + \frac{2cB_o\bar{R}}{T^3} \qquad [1.16g]$$

$$\delta' = -\frac{2bcB_o\bar{R}}{T^3} \qquad [1.16h]$$

$$c_p^o = \bar{A} + \bar{B}T + \bar{C}T^2 + \bar{D}T^3 \qquad [1.16i]$$

THE INTEGRATION

In order to see more clearly how to solve the set of Equations [1.1] to [1.16i], imagine reducing that system by means of substitution to three equations.

The first equation can be obtained by substituting C (Equation [1.2]), H (Equation [1.3]), and $\frac{dH}{dt}$ (Equation [1.3] differentiated with respect to time) into Equation [1.1]. The equation resulting from these substitutions is a relationship (actually Gilmore's bubble wall equation) between R, $\dot{R}$, $\ddot{R}$, P, and $\dot{P}$ which can be solved explicitly for $\ddot{R}$, i.e.,

$$\ddot{R} = G\,(R, \dot{R}, P, \dot{P}) \qquad [2.1]$$

The second equation is Equation [1.8], the Beattie-Bridgeman equation of state, after substituting $v_o\left(\frac{R}{R_o}\right)^3$ for v (the substitution $v = v_o\left(\frac{R}{R_o}\right)^3$ is obtained by solving Equation [1.6] for v). This new relationship between P, R, and T can be solved explicitly for T, i.e.,

$$T = B(R,P) \qquad [2.2]$$

The third equation constrains the gas to behave isentropically. To obtain it, first use Equations [1.16a] to [1.16i] in Equation [1.16] to get c_p as a function of v and T. Next substitute $c_p = c_p\,(v,T)$ into Equation [1.15]. Finally, after carrying out the indicated partial differentiation with the aid of Equation [1.8], replace v in this new equation by $v_o\left(\frac{R}{R_o}\right)^3$ to get a differential relationship between P, R, T, i.e.,

$$c_p\Big(v(R),T\Big)\ dT = T\ \frac{\partial v(P,T,v(R))}{\partial T}\Big)_p\ dP$$

where

$$dP = f\ (R,\ T,\ P)dT \qquad [2.3]$$

The three Equations [2.1], [2.2], and [2.3] represent, respectively, Gilmore's bubble wall equation, the Beattie-Bridgeman equation of state, and the condition of zero entropy change in the gas ($ds = o$). The functions G, B, and f appearing in these equations, although very cumbersome, are known functions of their respective variables.

The system of Equations [2.1] to [2.3] can be further simplified by eliminating the variable T, temperature, between Equations [2.2] and [2.3],

$$T = B\ (R,P) \qquad [2.2]$$

$$dP = f\ (R,T,P)dT \qquad [2.3]$$

as follows: First replace T in f (R,T,P) by T = B (R,P), giving

$$\begin{aligned} dP &= f\ (R,B(R,P),P)dT \\ &= \phi\ (R,P)dT \qquad [2.4] \end{aligned}$$

Using Equation [2.2] again, dT can be found in terms of P, R, dP, and dR. Recalling the chain rule,

$$dT = \frac{\partial B}{\partial R}\ dR + \frac{\partial B}{\partial P}\ dP \qquad [2.5]$$

where $\frac{\partial B}{\partial R}$ and $\frac{\partial B}{\partial P}$ are known functions which can be found in terms of P and R; call these functions g and h, respectively.

Then

$$dT = g(R,P)dR + h\ (R,P)\ dP \qquad [2.6]$$

Now use Equation [2.6], the value of dT, in Equation [2.4] the result being,

$$dP = g(R,P)\ \phi(R,P)\ dR + h\ (R,P)\ \phi(R,P)dP \qquad [2.7]$$

Dividing Equation [2.7] by dt, gives an expression for $\frac{dP}{dt}$,

$$\frac{dP}{dt} = g(R,P)\ \phi(R,P)\ \frac{dR}{dt} + h(R,P)\ \phi(R,P)\ \frac{dP}{dt}$$

or using the dot convention to represent differentiation with respect to time,

$$\dot{P} = g(R,P)\ \phi(R,P)\dot{R} + h(R,P)\ \phi(R,P)\dot{P} \qquad [2.8]$$

Solving Equation [2.8] for $\dot{P}$, yields,

$$\dot{P} = \frac{g(R,P)\ \phi(R,P)\dot{R}}{1-h(R,P)\ \phi(R,P)} \qquad [2.9]$$

Now Equations [2.9] and [2.1] are a set of simultaneous ordinary differential equations in which R and P are the dependent variables and t is the independent variable.

Conceptually, Equations [2.1] and [2.9] are much easier to solve than Equations [1.1] to [1.6] because the former set of equations is a more compact representation. The actual solution of Equations [1.1] to [1.16i] need not involve a direct reduction by hand to the two Equations [2.1] and [2.9], however. Instead, this reduction can be reserved for the computer, but the reasoning behind such a reduction process is necessary in order to code the solution of Equations [1.1] to [1.16i] for the computer.

The numerical integration procedure, the method of Hamming, used to integrate Equations [2.1] and [2.9], applies only to systems of first order ordinary differential equations. Equations [2.1] and [2.9] were therefore modified by introducing the new variable U, where

$$\dot{R} = U \qquad [3.1]$$

Equations [2.1] and [2.9] then become, respectively,

$$\dot{U} = G\ (R,U,P,\dot{P}) \qquad [3.2]$$

and

$$\dot{P} = \frac{g(R,P)\ \phi(R,P)U}{1-h(R,P)\ \phi(R,P)} \qquad [3.3]$$

By imposing appropriate initial conditions, Equations [3.1] to [3.3] are numerically soluble by the method of Hamming. The general application of this method is discussed thoroughly in "Mathematical Methods for Digital Computers"[13] by Ralston and Wilf and is summarized briefly in a previous paper[3] by the writer.

INITIAL CONDITIONS

When all the thermodynamic characteristics of the gas and liquid have been determined, three initial conditions, R(0), U(0), and P(0), are required for the solution of Equations [3.1] to [3.3]. The initial situation is brought about by imagining that for all time prior to $t = 0$ there exists an infinite expanse of compressible liquid uniformly compressed to some pressure p_∞, and that at time $t = 0$ there suddenly appears in this liquid a nonpulsating spherical cavity of radius R(0) filled with some quantity of gas under a pressure P_o. Such an artificially conceived situation leads to some physically untenable consequences. For instance, if a point in the liquid is chosen such that it lies on the bubble wall at $t = 0$, then the pressure P at that point is

$$P = \begin{matrix} P_o & & t = 0 \\ & \text{for} & \\ p_\infty & & t < 0 \end{matrix}$$

By using Equations [1.1] to [1.3], Gilmore has shown that coincident with the appearance of the bubble there will be a relatively small inward jump in the velocity of the bubble wall, i.e., if an originally motionless gas-filled sphere is to obey Equations [1.1] to [1.3] for all $t \geq 0$ then it cannot suddenly appear without having an initial inward wall velocity at the instant it does appear. The approximate value of this velocity jump, $\dot{R}(0_+)$, obtained from Equations [1.1] to [1.3] is (see Gilmore's report for derivation)

$$U(0_+) = \dot{R}(0_+) = \frac{P_o - p_\infty}{\rho_\infty \quad c_\infty}$$

Associated with this jump is, of course, an infinite instantaneous acceleration of the bubble wall. In an effort to avoid the initial infinite acceleration, one may choose $\frac{P_o - p_\infty}{\rho_\infty \quad c_\infty}$ as the initial condition for U(0) and solve Equations [3.1] to [3.3] using initial conditions at $t = 0_+$ rather than at $t = 0$. This is exactly the approach taken in this report, i.e.,

$$U(0) = U(0_+) = \frac{p_o - p_\infty}{\rho_\infty \quad c_\infty}$$

THE EULERIAN VELOCITY AND PRESSURE FIELDS IN THE LIQUID

Provided the Euleran velocity is considerably less than the sound speed, an approximate method can be used to determine the Eulerian velocity and pressure at any standoff (given distance from the center of the bubble) in the liquid. The method was developed by Gilmore[4] and has been used and discussed in a previous paper[3] by the writer.

RESULTS

The equations appearing in the foregoing analysis have been coded in Fortran IV for the IBM 7090 digital computer to determine numerically the behavior of an imploding gas bubble in liquid both when the gas obeys the Beattie-Bridgeman equation of state and when the gas obeys the ideal gas law. Complete Fortran IV listings of computer programs based on both models can be found in Appendixes B and C. Data input instructions are included. Bubble radius, velocity, and pressure time histories calculated from these programs appear in Figures 1 to 13. Implosions involving several types of gases at various ambient and initial internal pressures are represented.

A comparison between the results obtained using the Beattie-Bridgeman equation and those obtained using the ideal gas law is made in Table 2. The influence which the kind of gas inside the bubble and its initial pressure have upon the peak collapse pressure is summarized in Figures 1 and 2 for depths from 100 to 20,000 feet of water. The kind of liquid which implodes on the gas also influences the peak collapse pressure as shown by Figure 3.

All the results can be extended to cases for spheres of any radius. Suppose that at depth h a solution exists for a sphere with initial radius R_o. The radius, velocity, acceleration, and pressure are known functions of time at the bubble wall and some standoff in the liquid. If the initial

(Text continued on page 27.)

TABLE 1

Constants for the Beattie-Bridgeman Equation of State, the Constant Pressure Heat Capacity Equation, Equation of the Ideal State, and γ Representing Various Gases

Gas	γ	Beattie-Bridgeman Constants: A_o $\frac{atm\ ft^6}{Mole^2}$	B_o $\frac{ft^3}{Mole}$	a $\frac{ft^3}{Mole}$	b $\frac{ft^3}{Mole}$	$C \times 10^6$ $\frac{ft^3\ {}^oR^3}{Mole}$	Region of Validity	$C_p^o = A + BT + eT^2 + DT^3$ when C_p^o is in BTU/lb mole ^oR or (closely) cal/g mole ^oK: A	$B \times 10^{-3}$	$C \times 10^{-6}$	$D \times 10^{-9}$	Region of Validity
Argon, A	1.668	331.20	0.6297	0.3729	0	5.596	-150-450 ^oC 114 atm	4.9690	0	0	0	273-5000 ^oK
Neon, Ne	1.667	54.528	0.330	0.3518	0	0.0944	-217-400 ^oC 106 atm	4.9650	0	0	0	273-5000 ^oK
Helium, He	1.667	5.542	0.2243	0.9585	0	0.00374	-252-400 ^oC 102 atm	4.9677	0	0	0	273-5000 ^oK
Hydrogen, H_2	1.405	50.57	0.336	-0.0811	-0.698	0.0471	50-850 ^oR	6.424	1.039	- 0.07854	0	273-3800 ^oK
Nitrogen, N_2	1.4	344.92	0.808	0.419	-0.111	3.92	50-850 ^oR	6.529	1.488	- 0.2271	0	273-3800 ^oK
Air	1.399	334.1	0.739	0.309	-0.716	4.05	230-850 ^oR	6.557	1.479	- 0.2148	0	273-3800 ^oK
Carbon Monoxide, CO	1.399	344.9	0.808	0.419	-0.111	3.92	230-850 ^oR	6.48	1.566	- 0.2387	0	273-3800 ^oK
Oxygen, O_2	1.396	382.53	0.741	0.410	0.0674	4.48	50-850 ^oR	6.732	1.505	- 0.1791	0	273-3800 ^oK
Ammonia, NH_3	1.304	613.91	0.547	2.729	3.062	445.6	50-850 ^oR	6.5646	6.1251	2.3663	-1.5981	273-1500 ^oK
Methane, CH_4	1.304	584.6	0.895	0.297	-0.254	11.98	490-850 ^oR	4.75	12.0	3.03	-2.63	273-1500 ^oK
Carbon Dioxide, CO_2	1.288	1,284.9	1.678	1.143	1.159	61.65	490-670 ^oR	C_p = 18.036 - 00004474T - 158.08 T 1/2				273-3800 ^oK
Propane, C_3H_8	1.128	305.8	2.9	1.173	0.688	112.12	490-850 ^oR	-0.966	72.79	-37.55	7.58	273-1500 ^oK
Butane, C_4H_{10}	1.094	456.5	3.944	1.348	1.51	327.02	490-850 ^oR	0.945	88.73	-43.8	8.36	273-1500 ^oK

TABLE 2

Comparison of Calculated Peak Internal Gas Bubble Pressures (PSI) Based on Beattie-Bridgeman and Ideal Gas Models

Gas	γ	Water Depth of Implosion (Ft): 100 BB	100 IG	500 BB	500 IG	1000 BB	1000 IG	3000 BB	3000 IG
Argon	1.668	255	255	4330	4330				
Neon	1.667	255	255	4330	4330				
Helium	1.667	255	255	4330	4330				
Nitrogen	1.4	307	306	8987	8200	35600	34000	186000	210000
Ammonia	1.304	342	339						
Methane	1.304	343	339						
Propane	1.128	451	450						
Butane	1.094	490	484						

BB = Beattie-Bridgeman Model

IG = Ideal Gas Model

Figure 1 - Peak Internal Bubble Pressure as a Function of Water Depth Showing the Trend in P_o for Various Values of γ

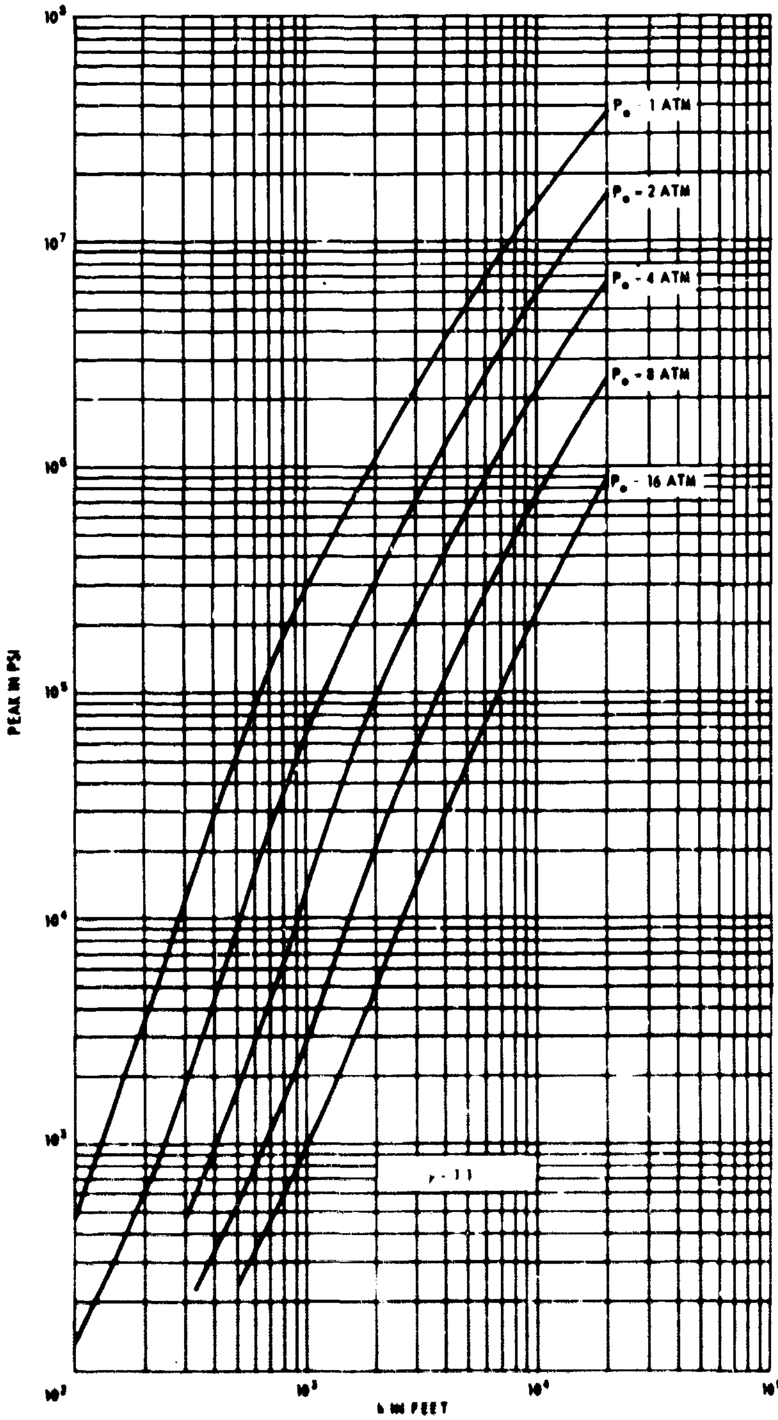

Figure 1a

Figure 1 (Continued)

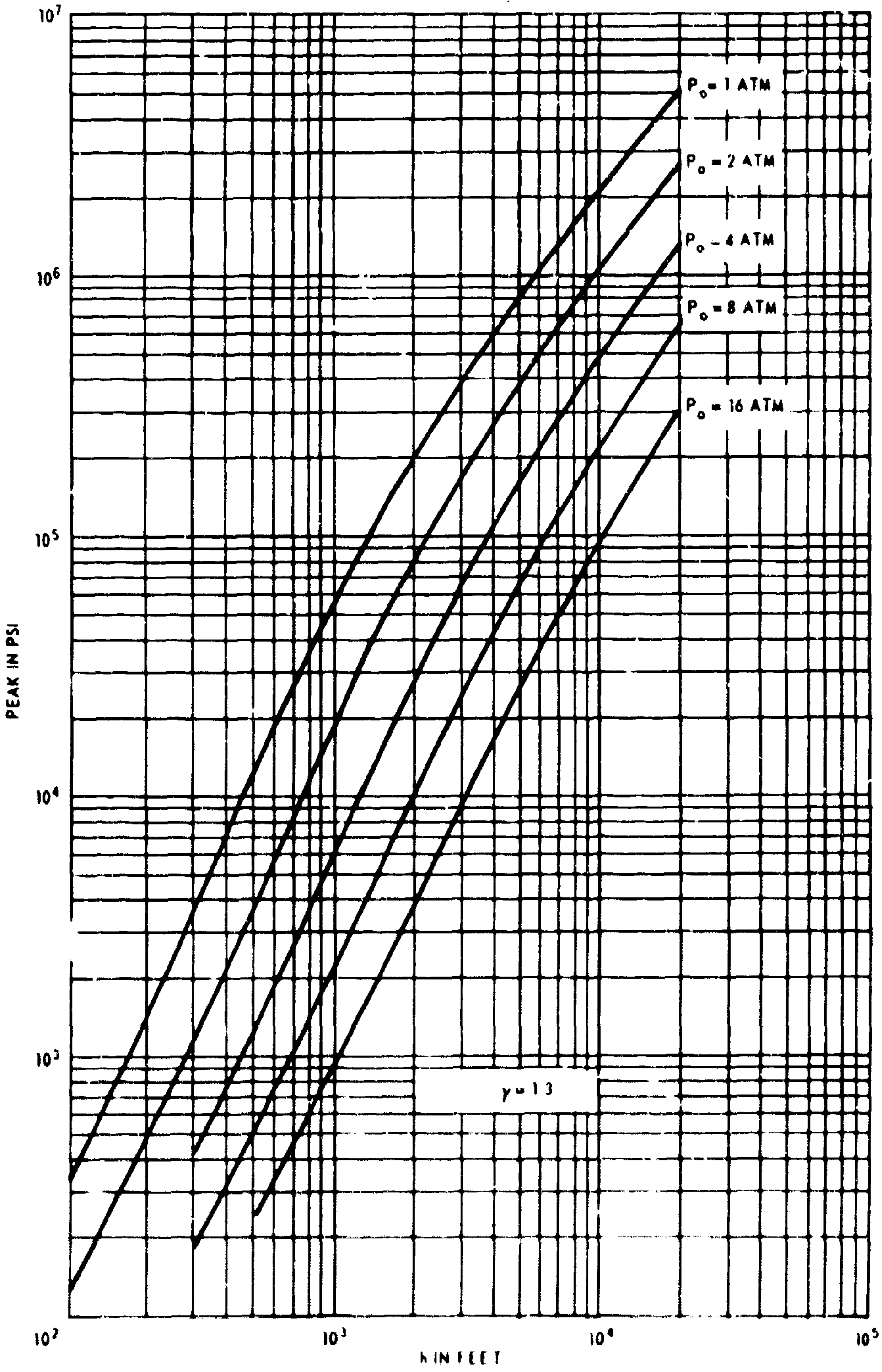

Figure 1b

Figure 1 (Continued)

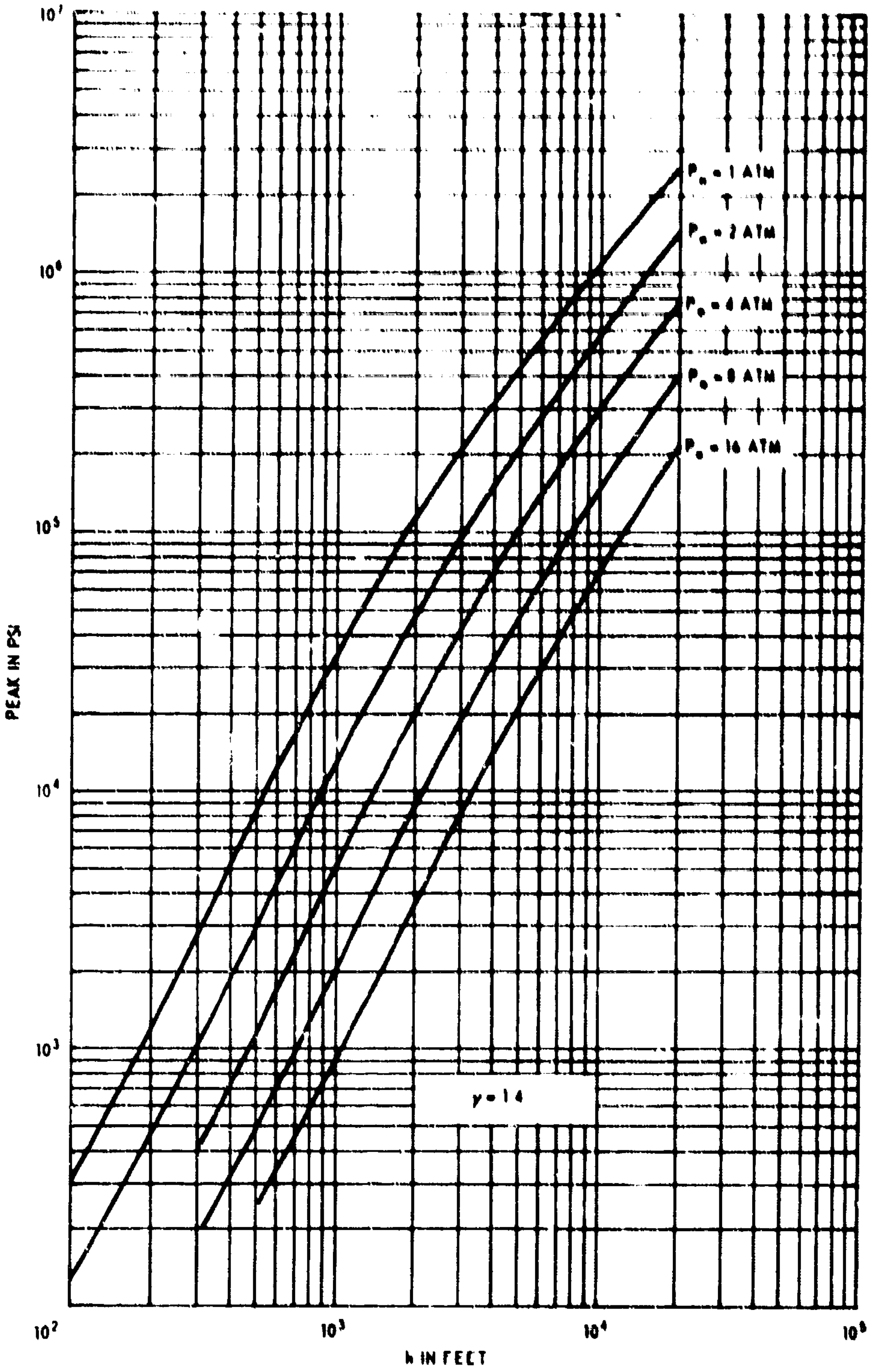

Figure 1c

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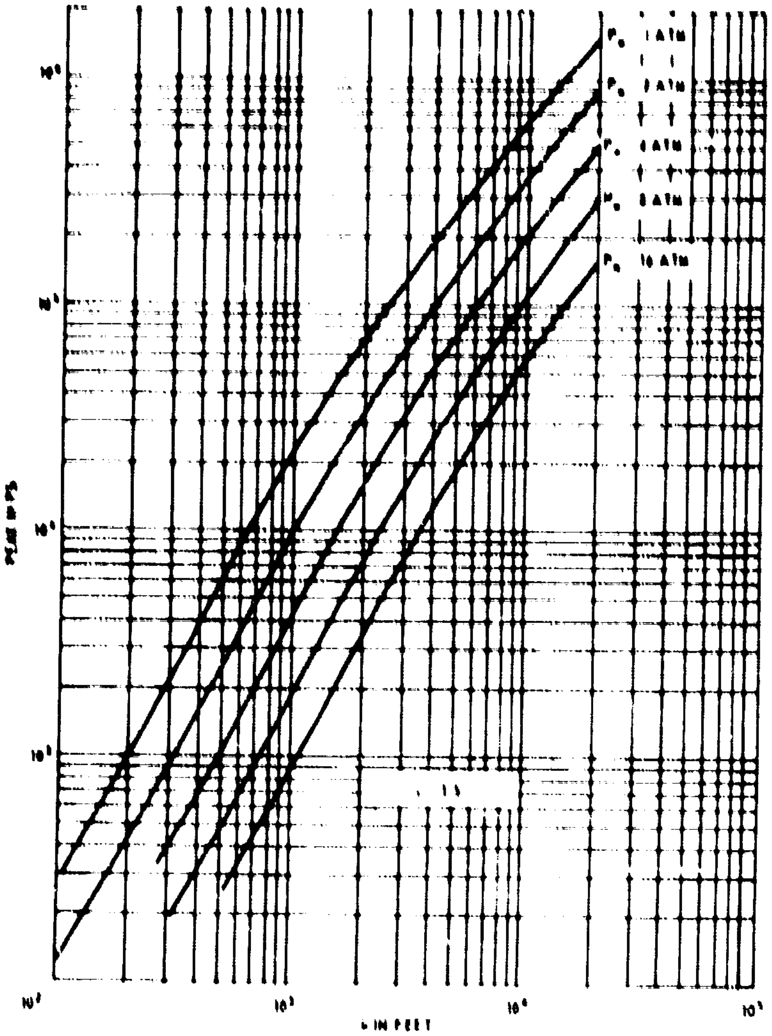


Figure 1d

Figure 1 (Continued)

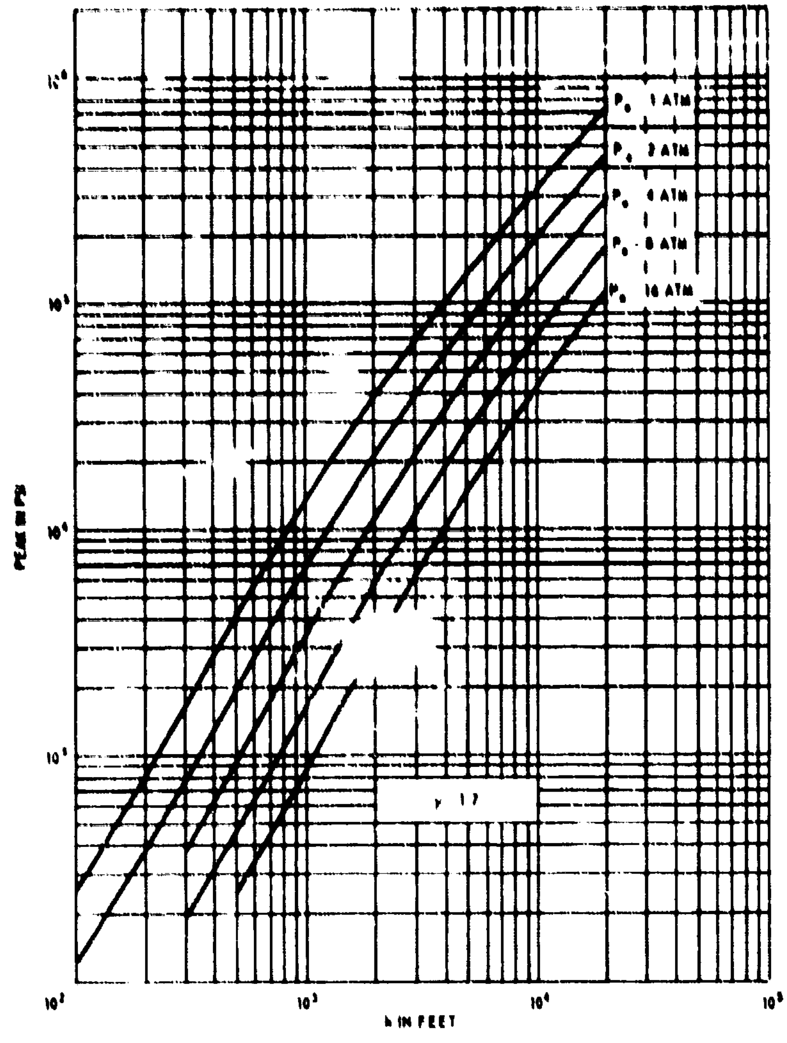


Figure 1e

Figure 2 - Peak Internal Bubble Pressure as a Function of Water Depth Showing the Trend in γ for Various Values of P_o

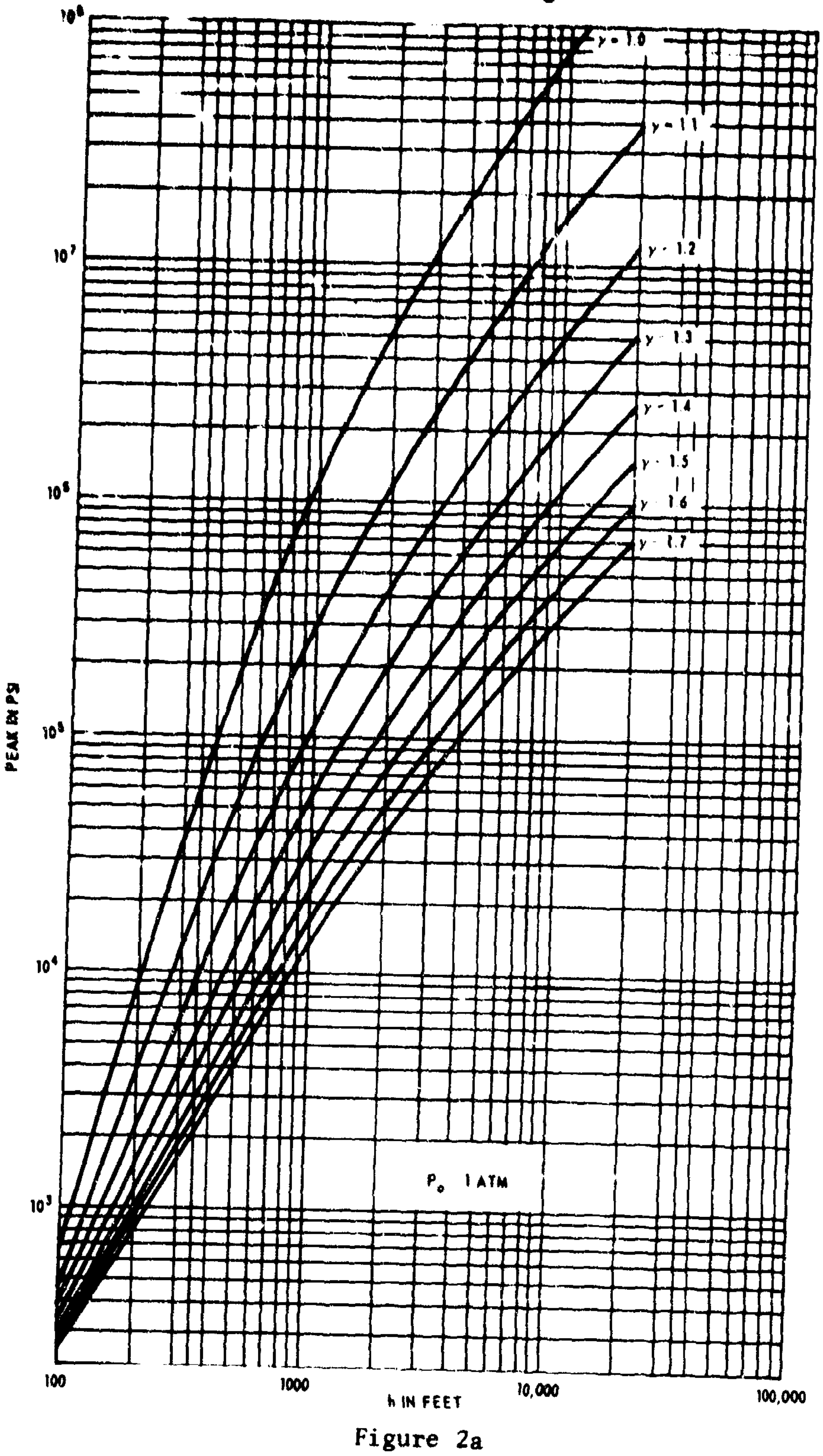


Figure 2a

Figure 2 (Continued)

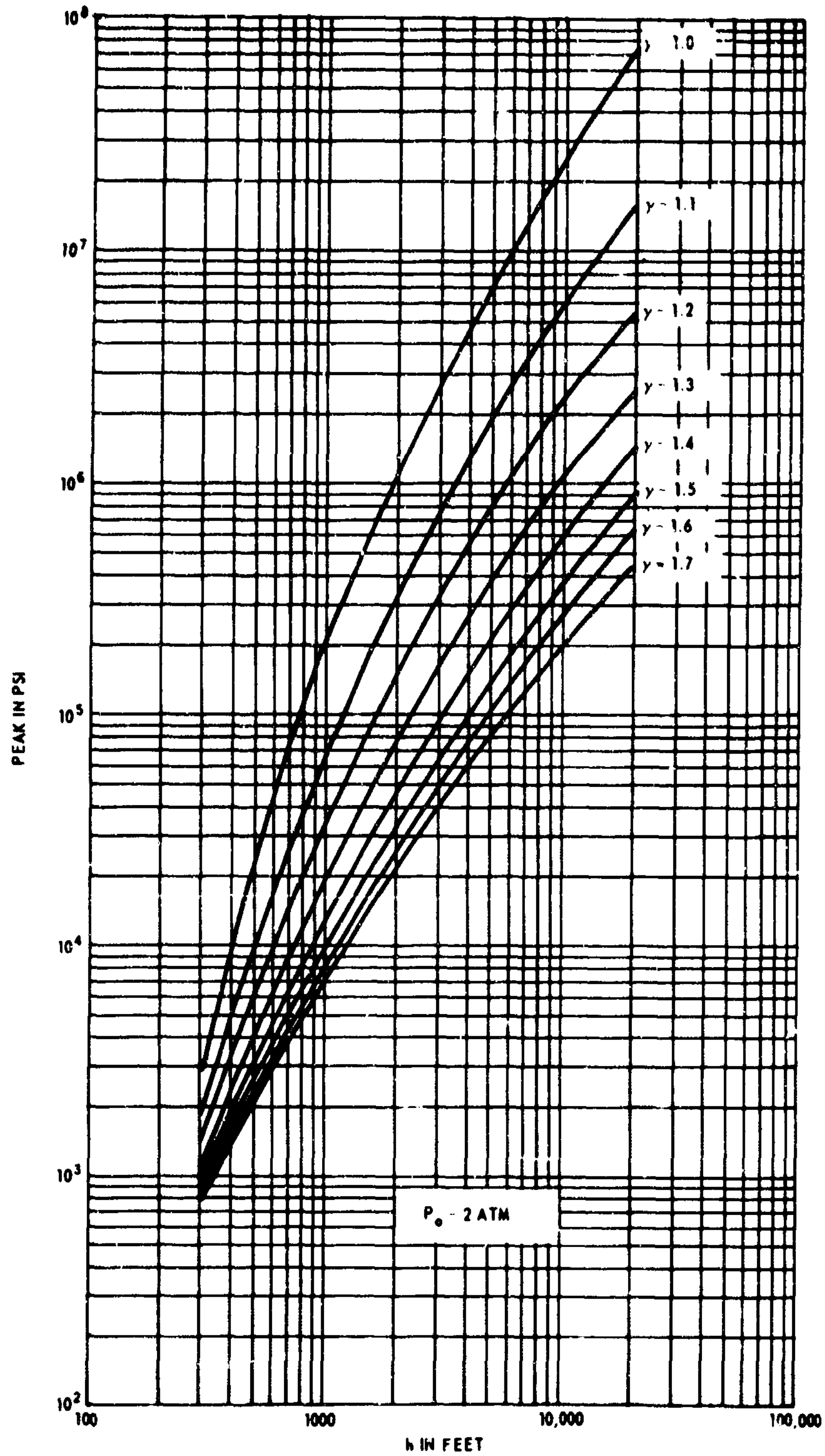


Figure 2b

Figure 2 (Continued)

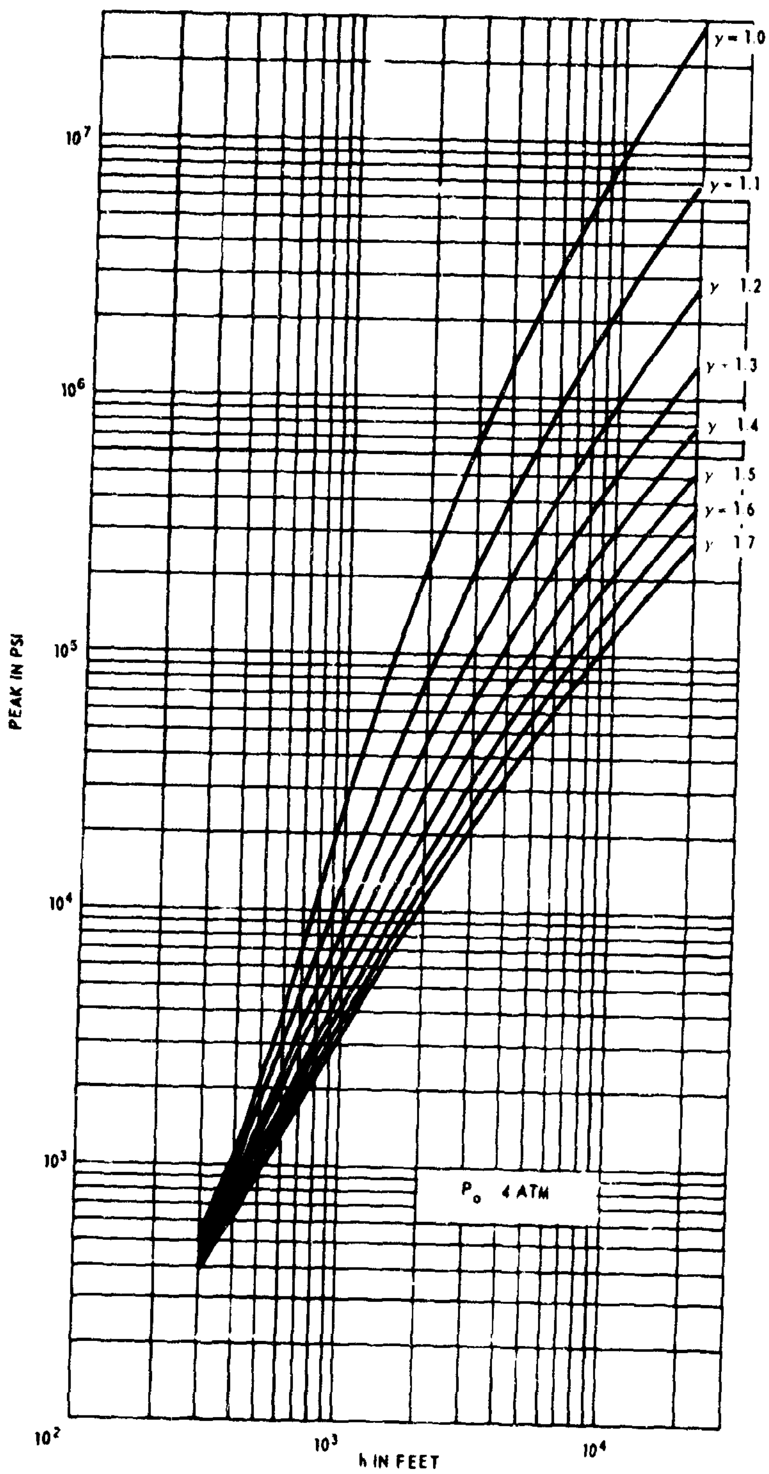


Figure 2c

Figure 2 (Continued)

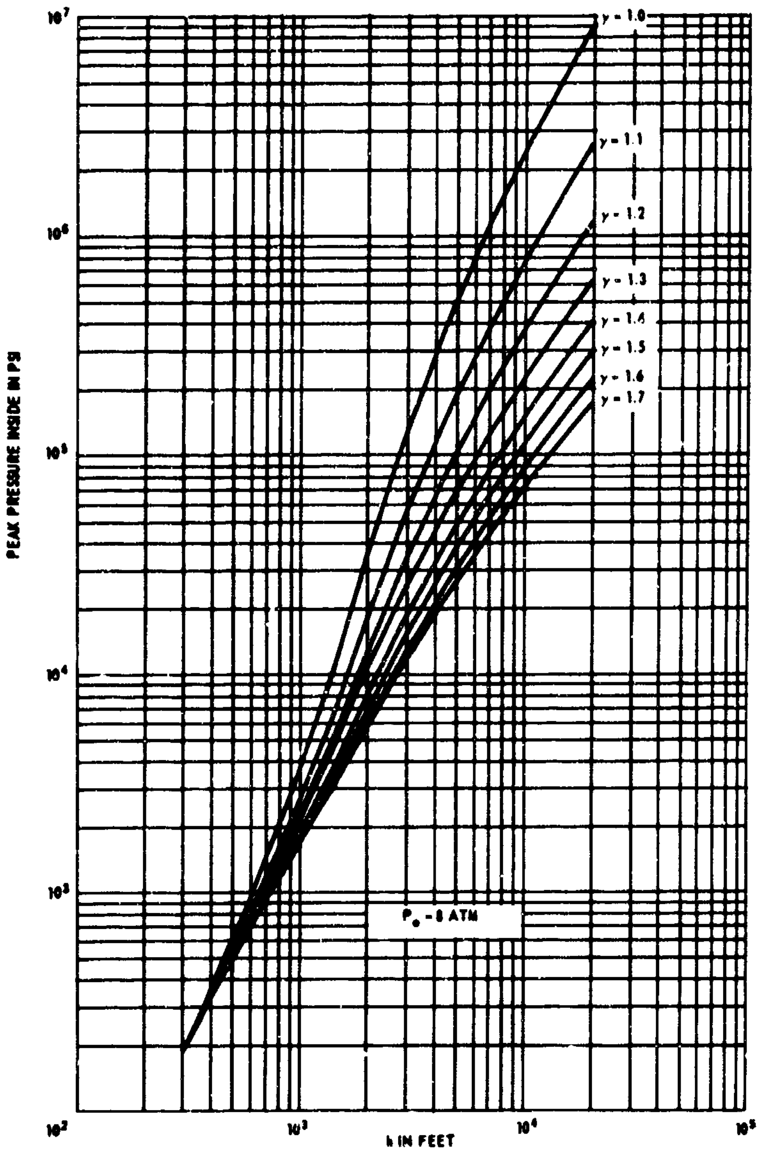


Figure 2d

Figure 2 (Continued)

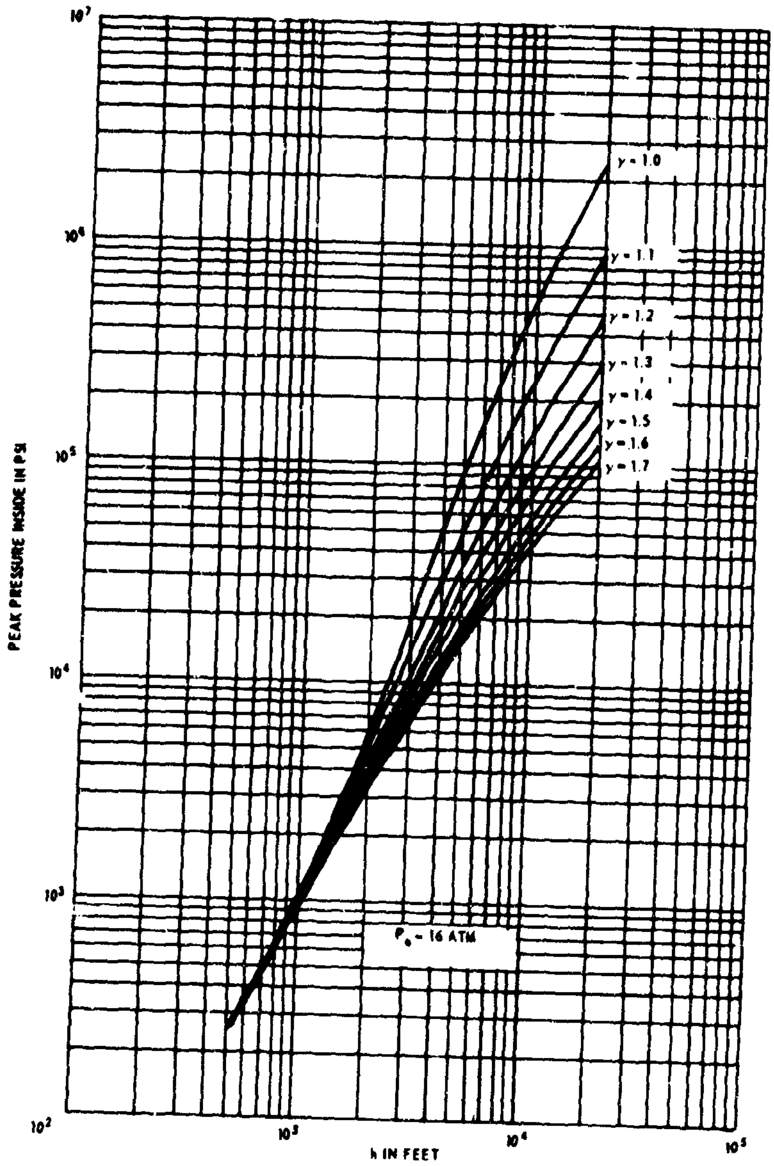


Figure 2e

Figure 3 - Peak Internal Bubble Pressure as a Function of Water Depth Showing the Trend in n for Various Values of B when P_o = 1 atm and γ = 1.4

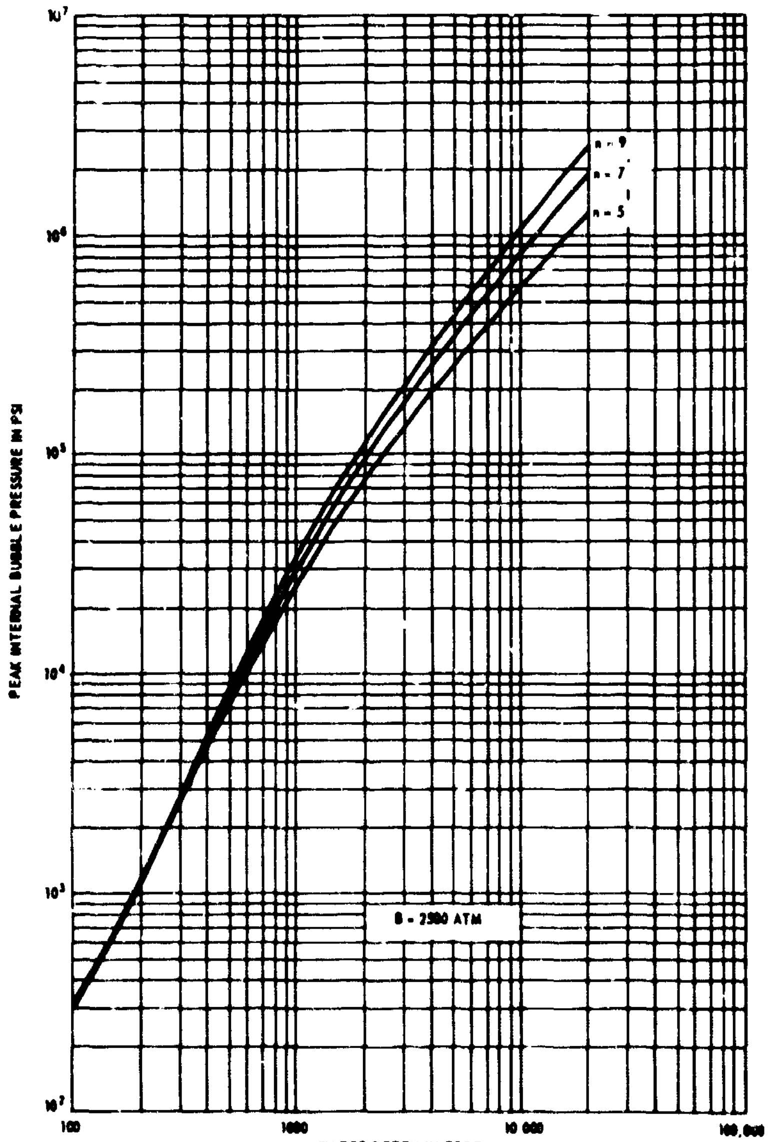


Figure 3a

Figure 3 (Continued)

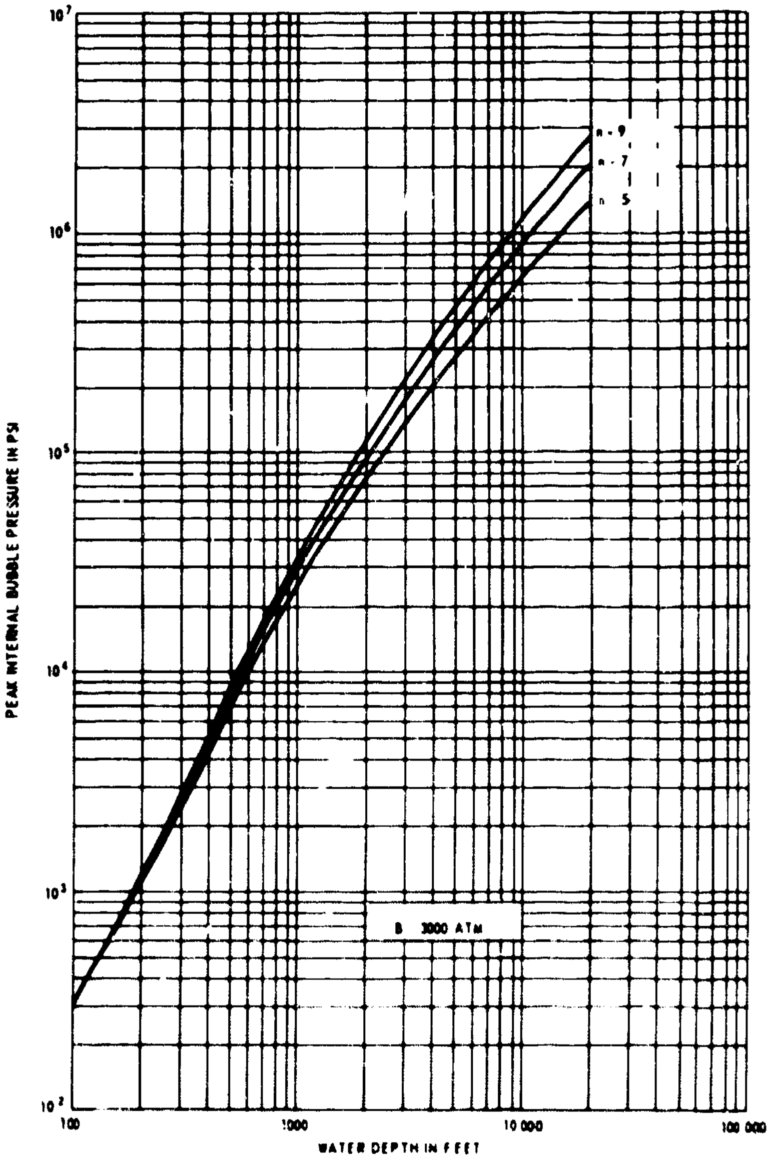


Figure 3b

Figure 3 (Continued)

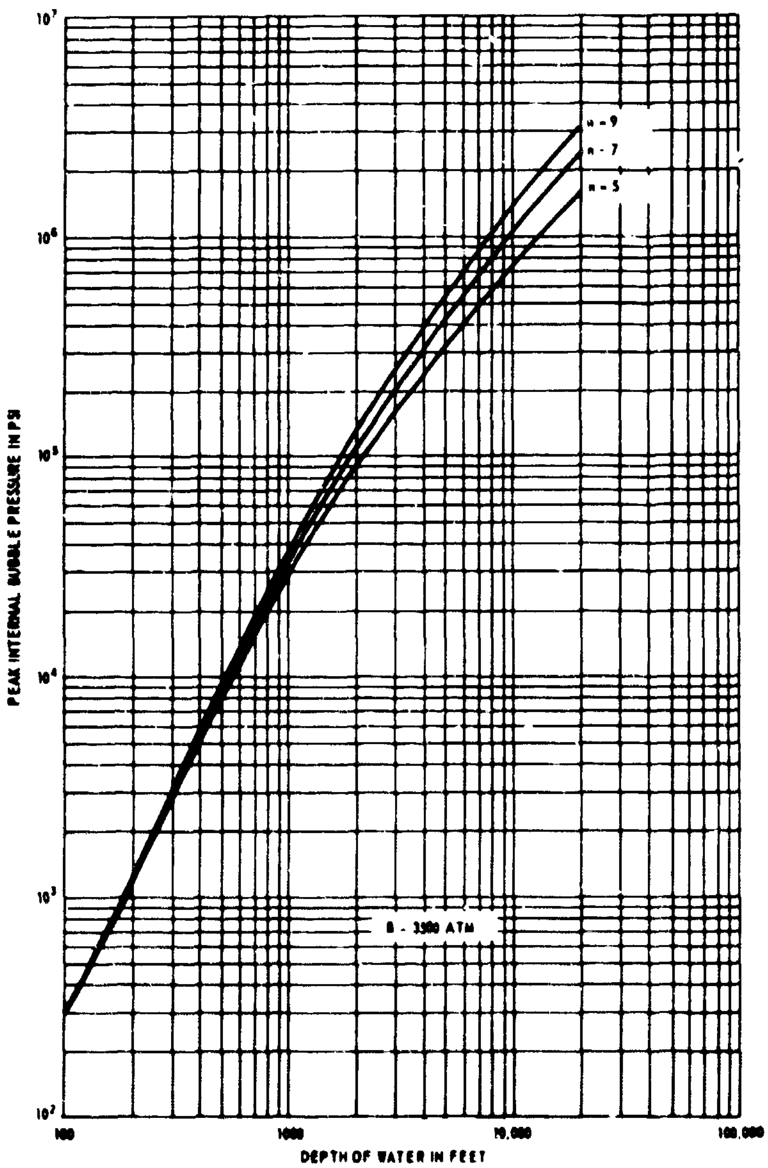


Figure 3c

radius is multiplied by λ = constant, then pressure and velocity will remain the same if radius, time, and standoff are multiplied by λ and acceleration is divided by λ.

DISCUSSION

Since the Beattie-Bridgeman equation accurately distinguishes between various types of gases, it provides a means for determining the influence which the type of gas originally inside the bubble has on the collapse. In an effort to discover the nature of this influence, the numerical analysis employing the Beattie-Bridgeman equation was carried out for implosions of gas bubbles filled with argon, neon, helium, nitrogen, ammonia, methane, propane, and butane. In each case the liquid was water, the initial sphere radius was 1.5 inches, and the initial internal gas pressure was 14.7 psi. It can be seen from Table 1 or 2 that these gases represent values of γ ranging from 1.668 to 1.094. Results for argon, nitrogen, and butane are shown in Figures 4 to 7 for depths of 100, 500, 1000 and 3000 feet of water. Peak internal gas bubble pressures are given in Table 2.

Unfortunately, the extent to which an analysis of this kind can be carried is seriously limited. Although the thermodynamic equations, the Beattie-Bridgeman equation, and the constant pressure heat capacity equation of the ideal state, are representative of gases at very high temperatures and pressures, these are quite often not as high as the values reached in the final stages of gas bubble implosions. The range of applicability of the thermodynamic equations depends upon the constants given for each particular gas (Table 1). Nitrogen, for which the equations are applicable for pressures up to 15,000 atm, is the exception rather than the rule. For example, the pressures and temperatures developed inside a gas bubble during the final stages of collapse at a water depth of 500 feet lie outside the range of applicability of the Beattie-Bridgeman equation when the bubble contains butane, propane, methane, or ammonia. At a depth of 1000 feet the thermodynamic equations are applicable only to the bubble containing nitrogen. In addition to not being applicable at very high temperatures and pressures, the Beattie-Bridgeman equation does not hold near the critical point. In none of the cases studied, however, was the critical point reached.

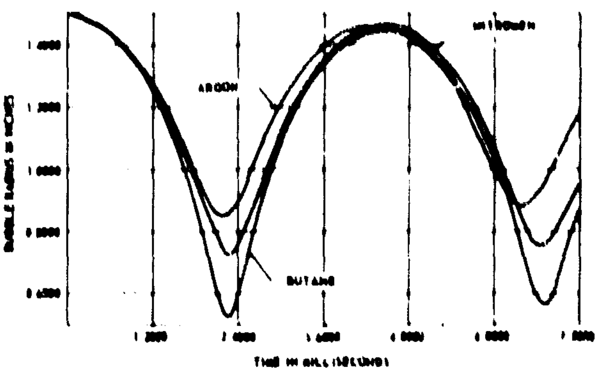
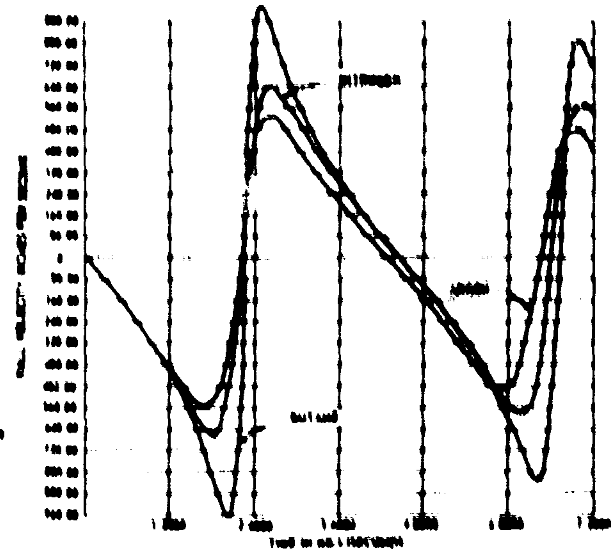


Figure 4a



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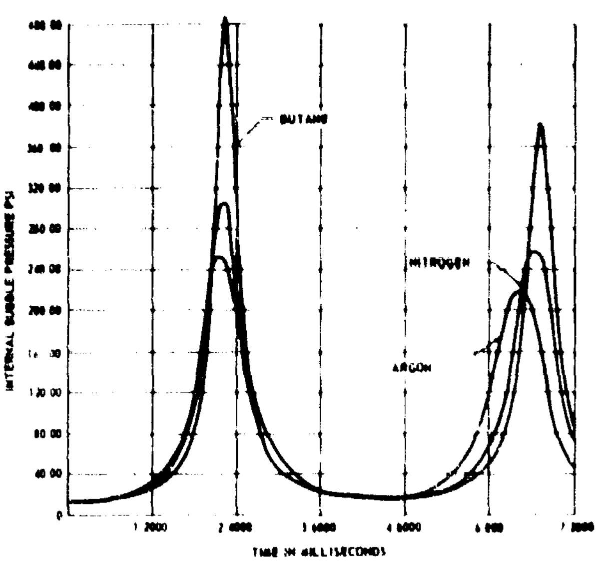


Figure 4c

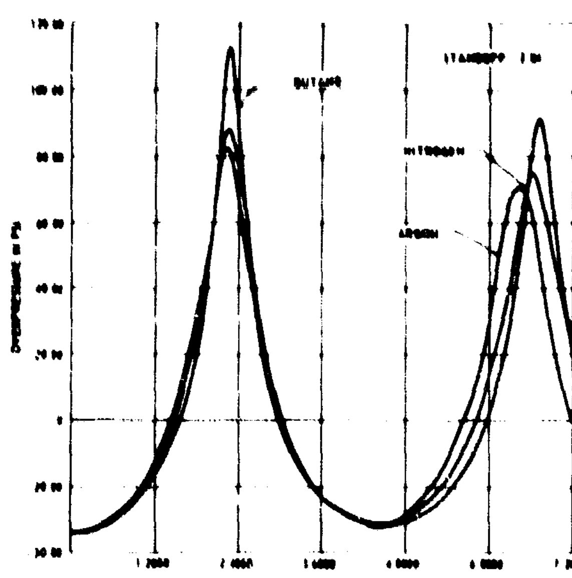


Figure 4d

Figure 4 - Beattie-Bridgeman Analysis of the Collapse of a 1.5 Inch Radius Bubble Filled with Butane/Nitrogen/Argon at 1 Atmosphere and 520° Rankine and Immersed in Water at a Depth of 100 Feet

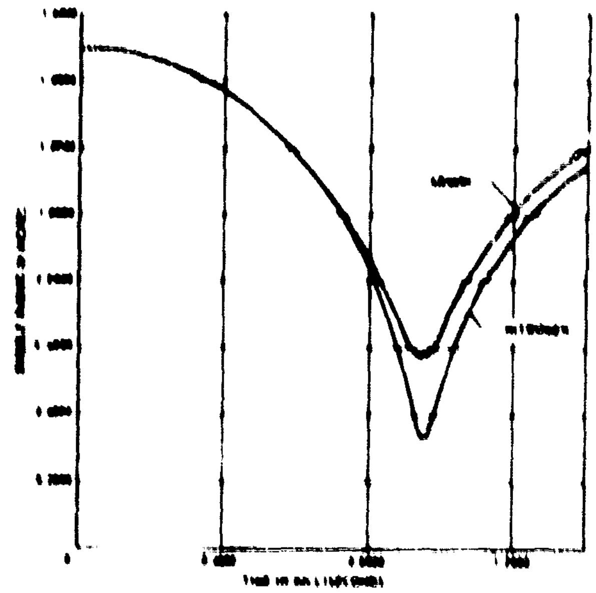


Figure 5a

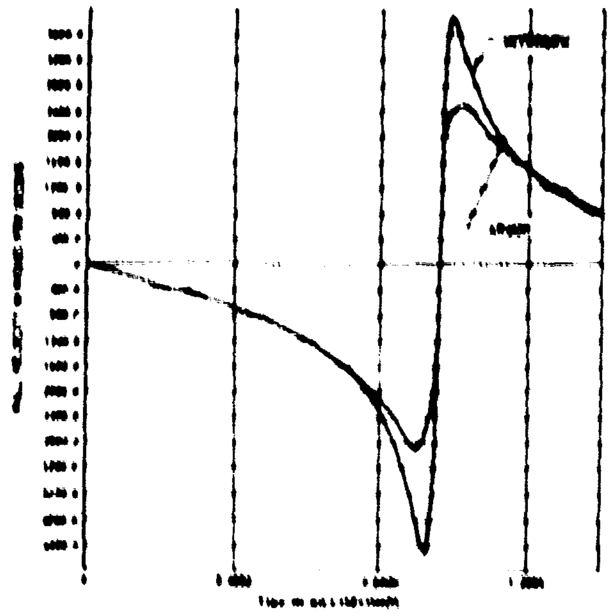
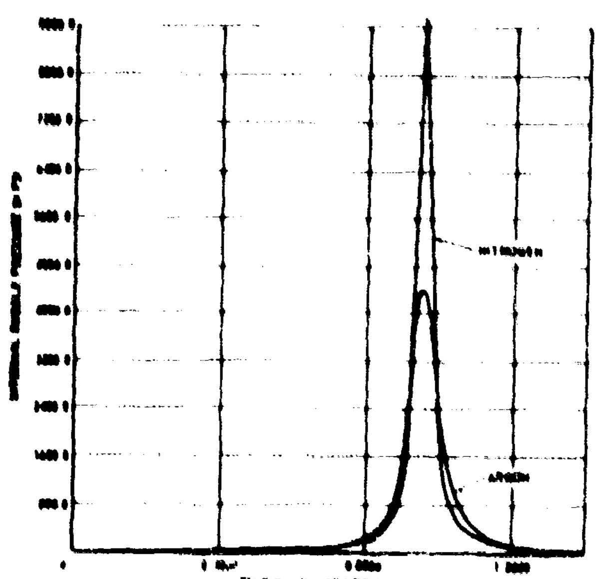


Figure 5b



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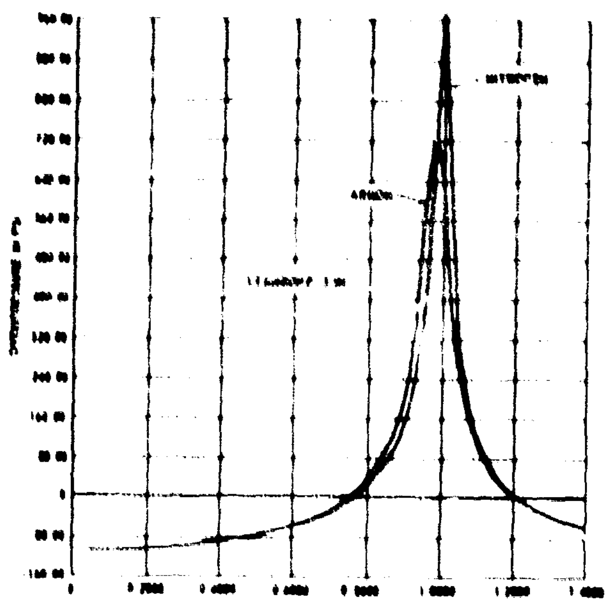


Figure 5d

Figure 5 - Beattie-Bridgeman Analysis of the Collapse of a 1.5 Inch Radius Bubble Filled with Argon/Nitrogen at 1 Atmosphere and 520° Rankine and Immersed in Water at a Depth of 500 Feet

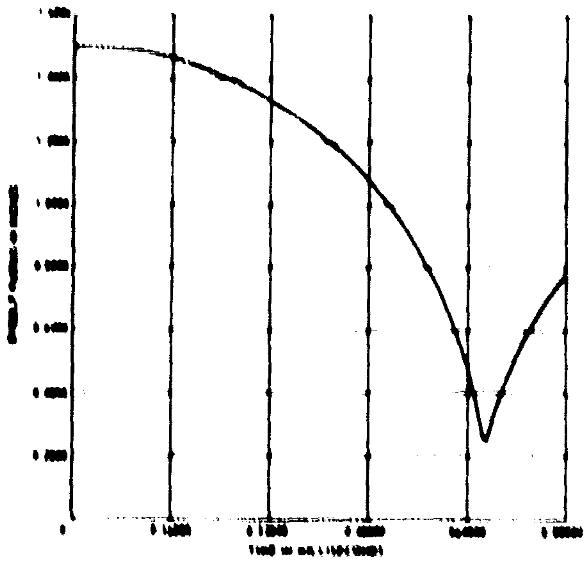
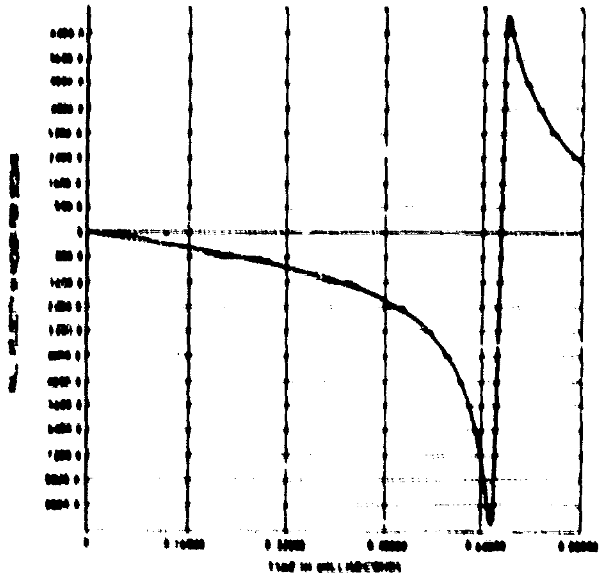


Figure 6a



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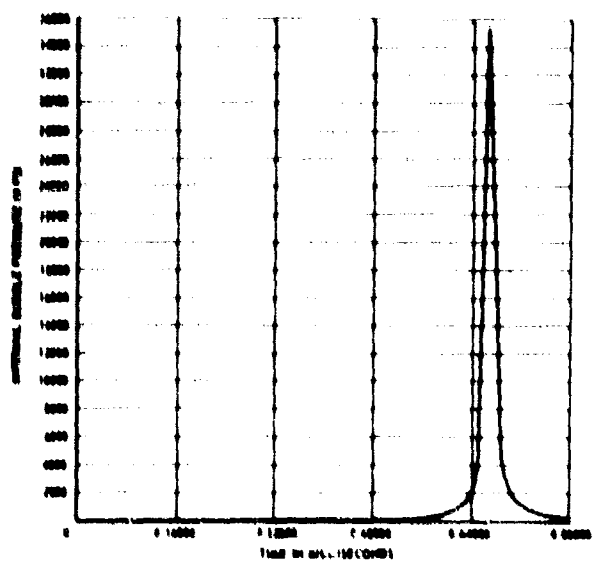

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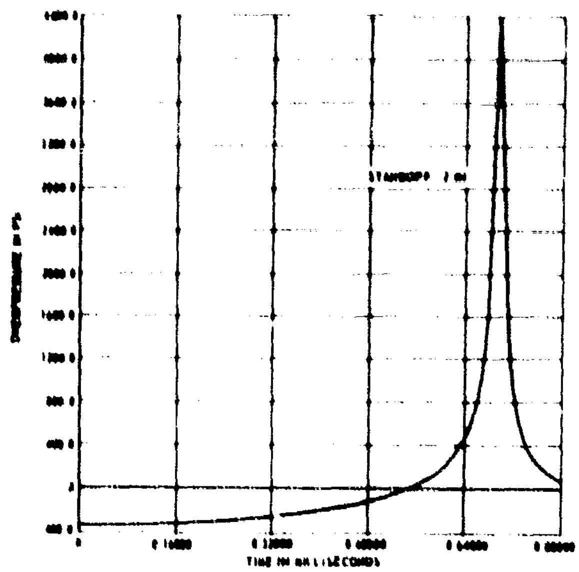

Figure 6d

Figure 6 - Beattie-Bridgeman Analysis of the Collapse of a 1.5 Inch Radius Bubble Filled with Nitrogen at 1 Atmosphere and 520° Rankine and Immersed in Water at a Depth of 1,000 Feet

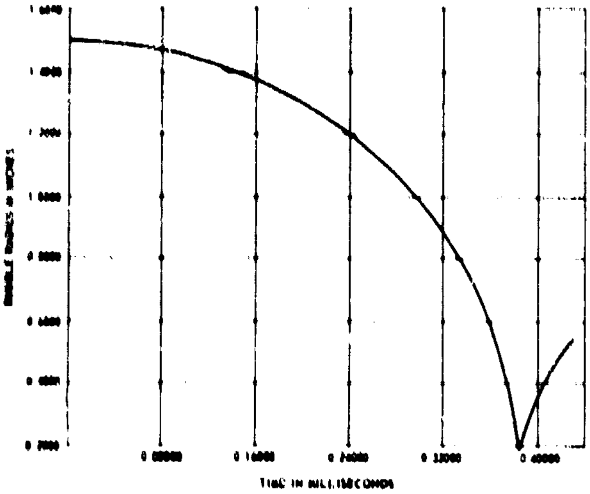

Figure 7a

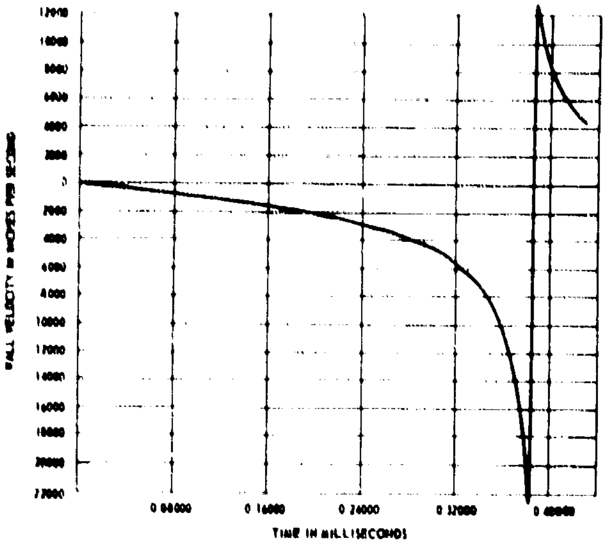

Figure 7b

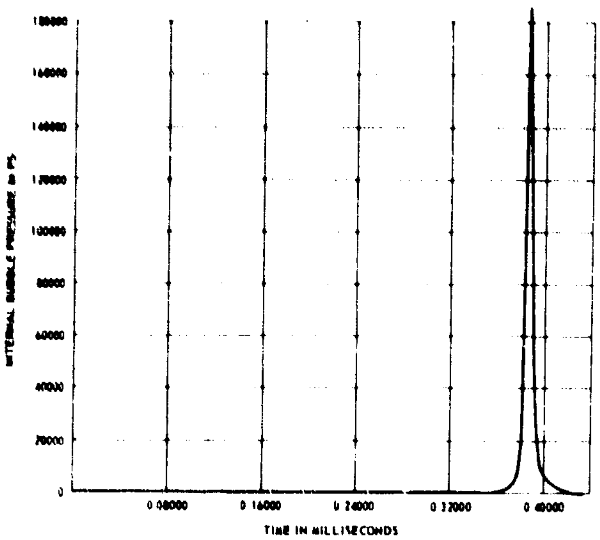

Figure 7c

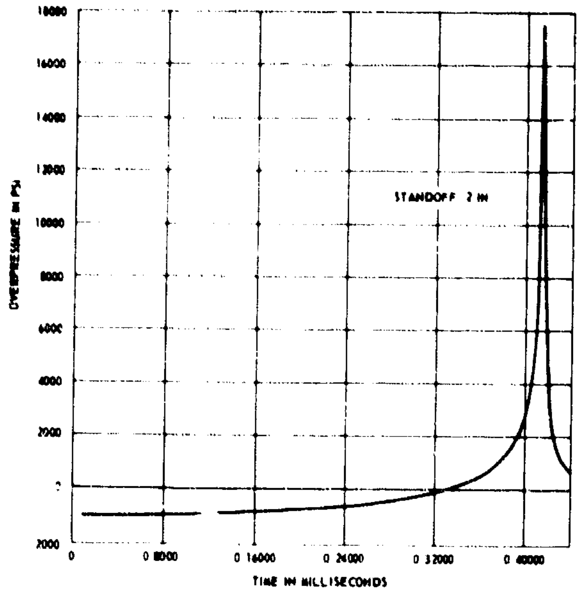

Figure 7d

Figure 7 - Beattie-Bridgeman Analysis of the Collapse of a 1.5 Inch Radius Bubble Filled with Nitrogen at 1 Atmosphere and 520° Rankine and Immersed in Water at a Depth of 3,000 Feet

Nevertheless, even at relatively shallow depths, it is clear from Figures 4 and 5 and Table 2 that the type of gas upon which the liquid implodes significantly affects the pressure developed during the final stages of implosion. Moreover, at each depth, the peak gas pressure developed inside the bubble is a monotone decreasing function of the value of γ for the gas. In fact, this phenomenon is so well characterized by the value of γ, that the behavior of an argon ($\gamma = 1.668$) gas bubble implosion, described by the Beattie-Bridgeman model, is essentially indistinguishable from that of a neon ($\gamma = 1.667$) or a helium ($\gamma = 1.667$) implosion under the same circumstances even though there are large differences among the Beattie-Bridgeman constants for these gases. Aside from the practical significance, this suggests that the ideal gas law for adiabatic behavior can be used to determine the influence of different gases on the peak pressure of collapse. As can be seen from Table 2, the ideal gas law agrees quite well with the Beattie-Bridgeman equation in describing the behavior of gases inside imploding bubbles in liquid.

The ideal gas model was used not only to study the same implosions studied by means of the Beattie-Bridgeman model, but also to extend the results obtained with the Beattie-Bridgeman model at low depths to greater depths. Excluding compressibility charts, the ideal gas law is the only practical means of establishing an equation of state for gases at those temperatures and pressures developed during implosions at great depths. The ideal gas model can be used to extend results to 30,000 feet, but it should be noted that these results are purely hypothetical for many gases. Beyond those depths at which the Beattie-Bridgeman model (with constants given in Table 1) can be applied, chemical reactions such as dissociation and ionization (which violate the condition of zero entropy change inside the bubble) may be expected to have significant effects on the collapse. The information summarized in Figures 1 and 2 is based on ideal gas behavior. A more detailed description of the effect of initial internal pressure at depths of 1000 and 10,000 feet is given in Figures 8 and

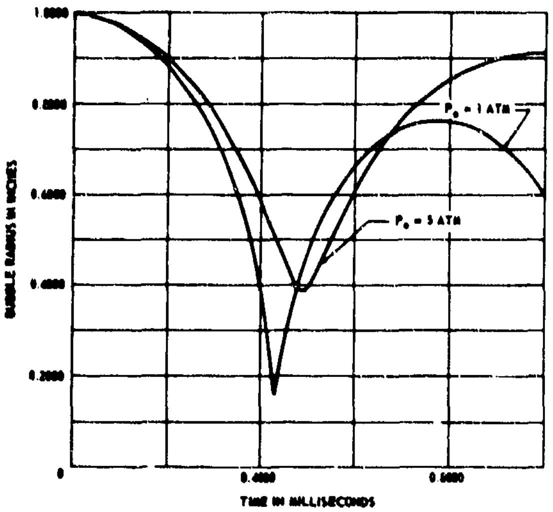

Figure 8a

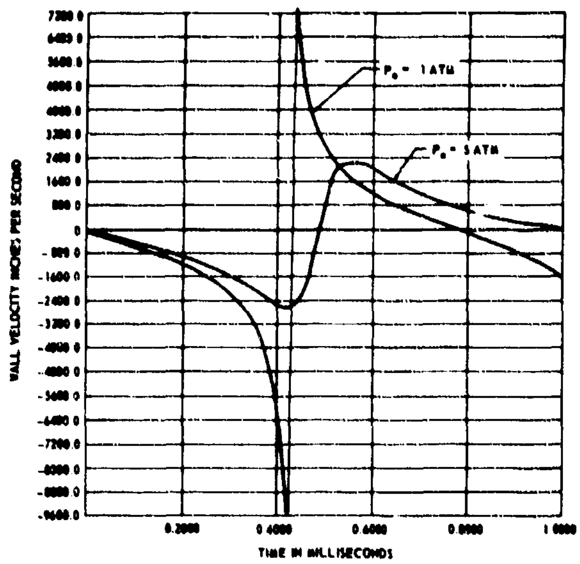

Figure 8b

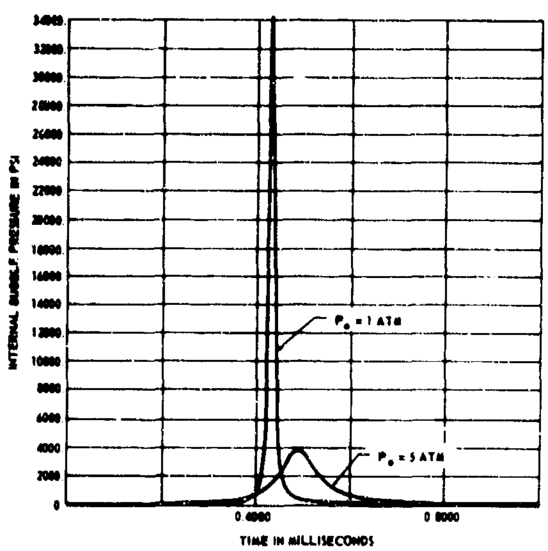

Figure 8c

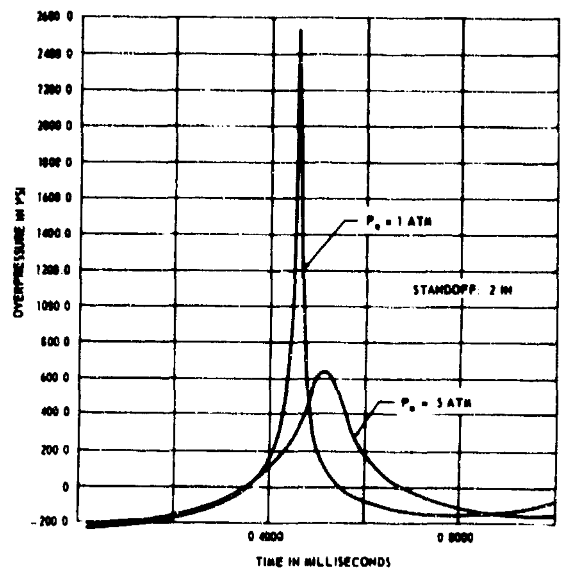

Figure 8d

Figure 8 - Comparison of 1000 Foot Water Depth Implosions of Spheres Filled to Pressures of 1 Atmosphere and 5 Atmospheres with a Gas Whose γ-Value is 1.4

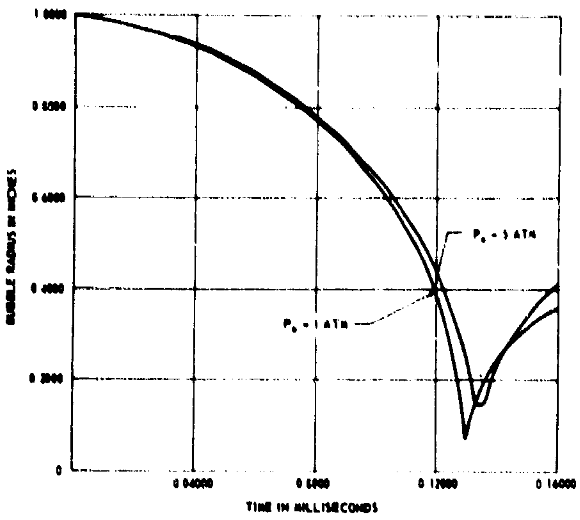

Figure 9a

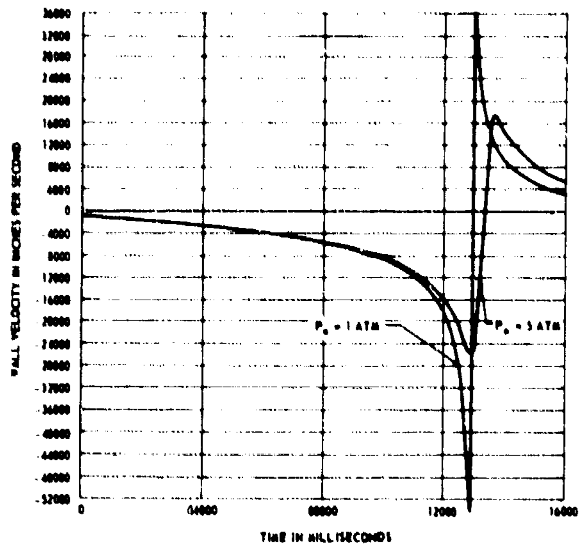

Figure 9b

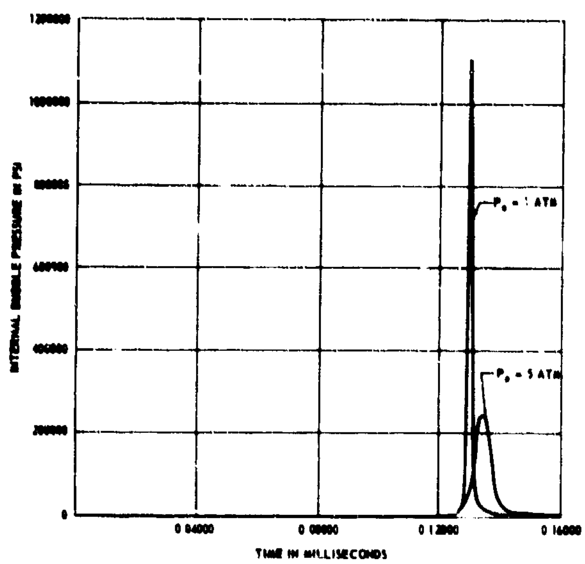

Figure 9c

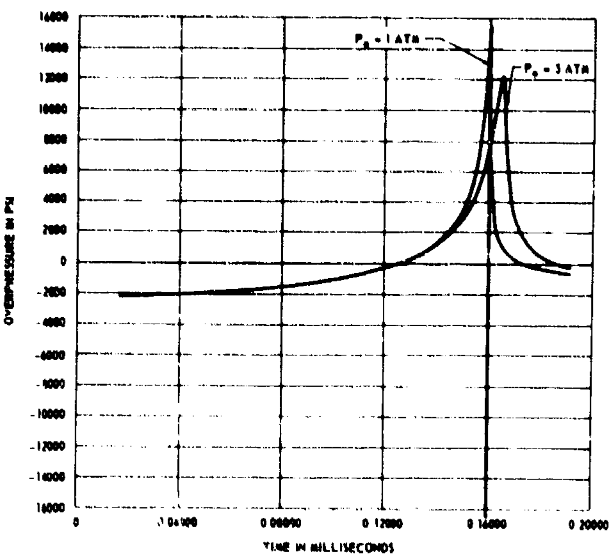

Figure 9d

Figure 9 - Comparison of 10,000 Foot Water Depth Implosions of Spheres Filled to Pressures of 1 Atm and 5 Atm with a Gas Whose γ-Value is 1.4

9* for nitrogen and initial gas pressures of 1 and 5 atmospheres. Likewise, for an initial internal pressure of 1 atmosphere, a more detailed description of the influence which the kind of gas inside the bubble exerts on the peak collapse pressure is given for argon and nitrogen in Figures 10 and 11* for water depths of 1000 and 10,000 feet.

The relationship between γ and the peak pressure developed in the gas bubble during collapse can be roughly explained in terms of the adiabatic compressibility of gases (the fractional change in volume of a gas in a reversible adiabatic compression). For any gas, the adiabatic compressibility is defined as

$$-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_S$$

If, for simplicity, ideal behavior (and constant γ) is assumed, then

$$Pv^{\gamma} = \text{constant}$$

for an adiabatic process. From this

$$-\frac{1}{v}\frac{dv}{dP} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_S = \frac{1}{P\gamma}$$

Thus the compressibility of a gas undergoing a reversible adiabatic process, such as a gas bubble collaps is inversely proportional to the value of γ associated with the gas. Note that it is also inversely proportional to the pressure of the gas. It follows that during the implosion those gases with relatively large values of γ are less compressible than those with

---

*The liquid overpressures plotted in Figures 9, 11, and 13 appear to have superimposed upon them a very sharp spike near their peaks. Similar sharp spikes appear in the corresponding Eulerian velocity plot. The spikes are actually not spikes, but points at which the pressures and velocities are multivalued. As Gilmore[4] has explained, these multivalues result from the catching up and overtaking of characteristics with other characteristics which originated earlier at the bubble wall. The different speeds of propagation of the characteristics are due to the changing sonic velocity of the liquid.

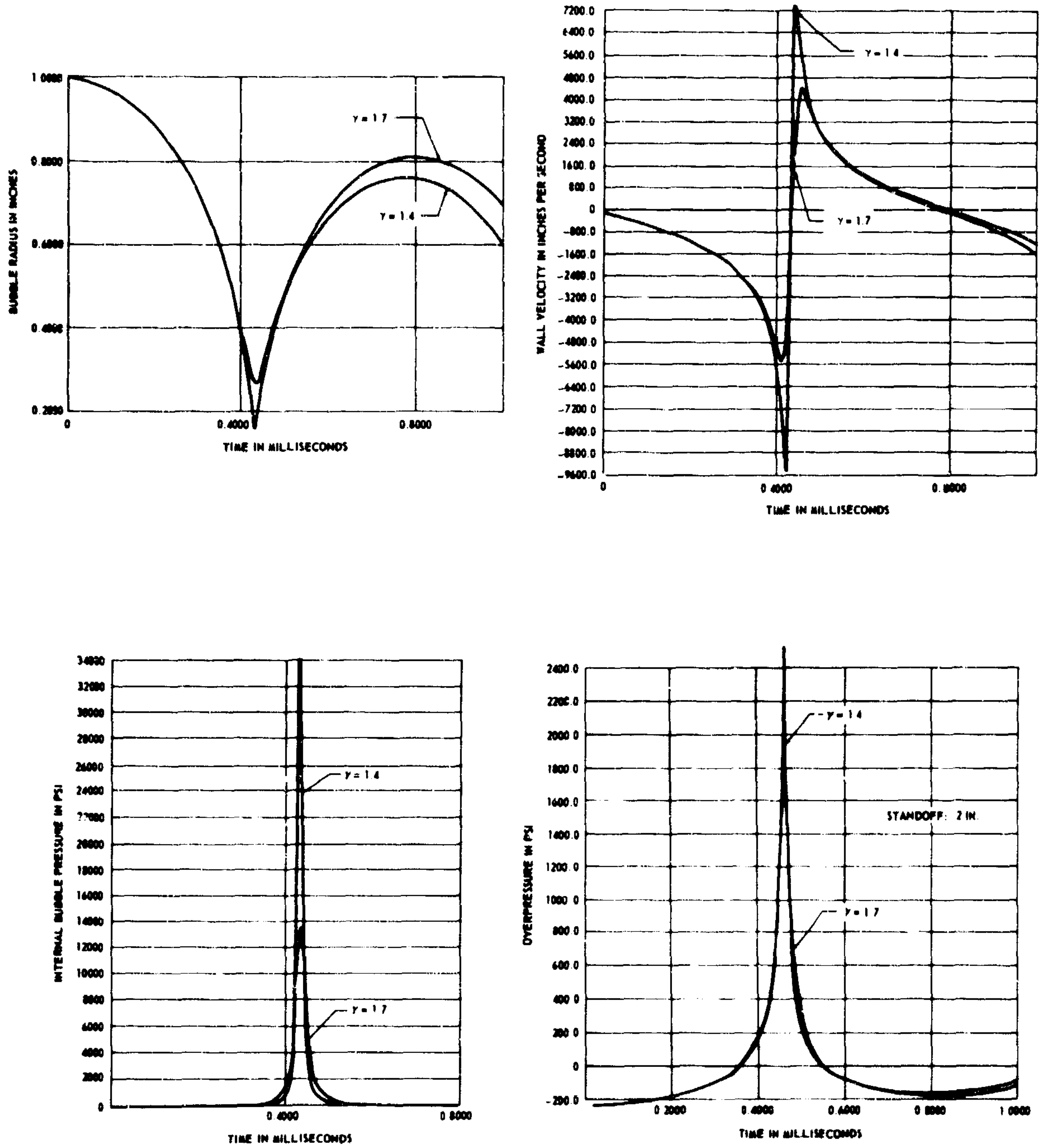

Figure 10 - Comparison of 1000 Foot Water Depth Implosions of Spheres Filled to a Pressure of 1 Atmosphere with Gases Whose γ-Values are 1.4 and 1.7

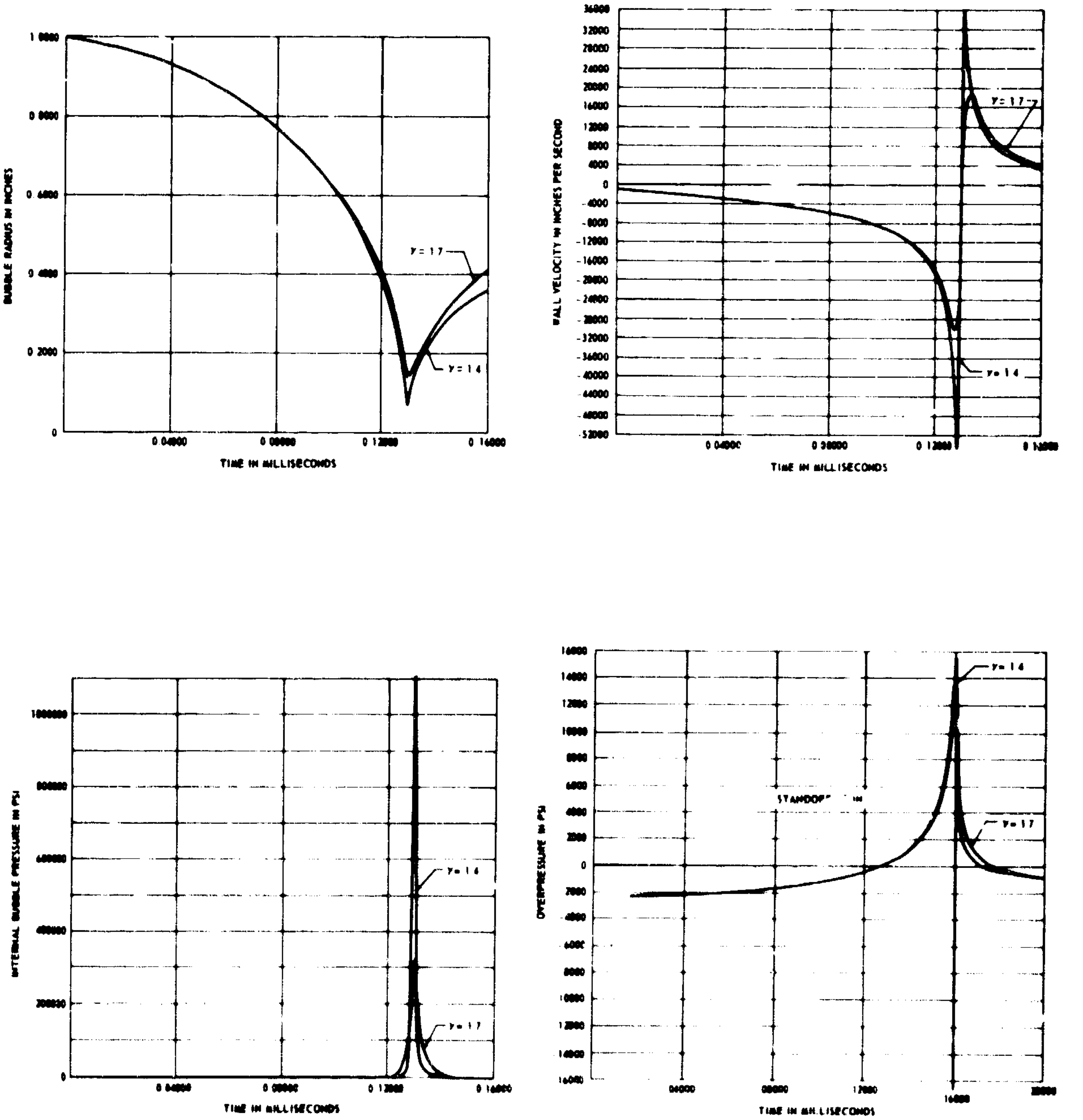

Figure 11 - Comparison of 10,000 Foot Water Depth Implosions of Spheres Filled to a Pressure of 1 Atm with Gases Whose γ-Values are 1.4 and 1.7

relatively small values, at the same pressure. At any given pressure, the relatively incompressible gases (neon, argon, helium) offer more resistance to the inrushing liquid than do the relatively compressible gases (butane, propane). This added resistance is offered by the relatively incompressible gases not only in the final stages of collapse, but also in the early and intermediate stages when even a very small decrease in the velocity of the inrushing liquid can greatly reduce the momentum, and thereby the pressure, in the final stage. Because of the nature of the mechanism described above, the peak collapse pressure can be reduced by increasing the value of γ, or, according to the idealized equation for adiabatic compressibility, by increasing the initial internal pressure of the gas inside the sphere.

It has so far been verified that a small decrease in the compressibility of the substance inside the bubble can bring about a considerable decrease in the peak pressure of collapse in water. This suggests a simple solution of the sympathetic implosion problem in which the liquid pressure field generated by the collapse of one glass sphere causes the failure and subsequent collapse of neighboring glass spheres. A solution would be to fill the spheres with a substance which is more incompressible than any gas. At the same time it is desirable to obtain maximum buoyancy so that the density of this substance should be at least comparable to that of gases. Unfortunately, no such substance exists. All solids and liquids are at least several orders of magnitude denser than gases. Thus, because of minimum buoyancy considerations, it is difficult to justify glass buoyancy spheres filled with anything other than a gas, unless weight compensation is provided by imbedding the spheres in a suitable material considerably lighter than water.

It is theoretically possible to decrease the peak pressure by a method other than that discussed above. This scheme utilizes the changing pressure and temperature inside the collapsing bubble to produce a chemical reaction involving gases. The gaseous reaction products would indirectly decrease the compressibility of the gas mixture by directly increasing the pressure in one, or a combination, of the following two ways:

1. Heat may be a product of the reaction. Since no heat is exchanged between the liquid and the gas mixture during collapse all of the heat energy generated by the reaction would go into raising the temperature of

the gas mixture above that to which it would normally be raised by compression alone. If the number of moles of product equals the number of moles of reactant, and if the behavior of the mixture roughly follows the ideal gas law

$$Pv = nRT$$

then the increase in temperature must be accompanied by an increase in pressure (and a slight increase in volume) to a value above that which it would assume if there were no reaction.

2. The total number of moles of the reacting gases may be less than the total number of moles of gases produced. The net result would be an increase in the total number of moles of gas mixture. Assuming that the heat of reaction is very small, and that the mixture does not deviate significantly from ideal behavior, i.e., if again

$$Pv = nRT$$

then the increase in the total number of moles of mixture must be accompanied by an increase in pressure (and a slight increase in volume) to a value above that which it would assume if there were no reaction.

During an actual chemical reaction in which all reactants and products are gases confined as in the bubble, the liberation or absorption of heat and the increase or decrease in the total number of moles of mixture generally occur simultaneously and tend to oppose each other to maintain constant pressure as the reaction proceeds. It is unlikely, however, that exactly constant pressure can be maintained. Inside an imploding gas bubble the net result of a chemical reaction may serve either to increase or decrease the pressure above or below that which it would normally be in the absence of a reaction. The reaction can then be favorable or unfavorable in arresting the collapse.

All possible types of such chemical reactions fall into two categories: reactions which proceed immediately (possibly explosively) to completion, and equilibrium reactions in which the extent to which the reaction proceeds is usually determined by the temperature and pressure of the mixture of reactants and products. It would appear at first that equilibrium reactions have a great advantage over explosive reactions. Since an explosion inside an imploding bubble would necessarily be triggered by the collapse of that bubble, the intensity of the explosion would be approximately the same regardless of the depth of implosion. If the magnitude of that intensity were set to arrest the implosion of a bubble at a water depth of 10,000 feet, than at a 100 foot water depth that magnitude of intensity would possibly be more devastating than a simple implosion without an internal explosion.

Following this reasoning, numerical calculations were made* to determine roughly the behavior at a 1000 foot water depth of collapsing bubbles filled with N_2O_4 and NO_2 in chemical equilibrium at a temperature of 500°R to pressures of 1/2, 1, and 2 atmospheres. This particular equilibrium reaction was chosen because its properties are well known. About 90 percent of the original gas mixture by weight consisted of N_2O_4 which is favored by low temperatures and high pressures. The results showed the reaction to be a perfect illustration of the opposing effects of heat of reaction and change of number of moles of mixture discussed previously. Since NO_2 is favored by high temperatures and low pressures and since the temperature and pressure of the mixture were simultaneously increased by the compression, the NO_2 was favored about as much as the N_2O_4.

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*The calculations were very similar to those made here for inert gases except that additional equations were necessary to determine the degree of dissociation of N_2O_4 and to correct for the entropy introduced inside the bubble by the reaction. The functional relationship between the constant pressure equilibrium constant for the mixture and the temperature of the mixture was based on the observations of Bodenstein.[19] The kinetics of the reaction were ignored because the equilibrium establishes itself quite rapidly. For initial internal pressures of 1/2 and 1 atmosphere, the peak collapse pressure was well above that which it would normally be if there were no reaction and if the γ-value of the gas were 1.7. Nevertheless, at an initial pressure of 2 atmospheres, the peak pressure was about 1/2 that of an implosion involving an inert gas having a γ-value of 1.7 and initial pressure of 2 atmospheres. The validity of the equation for equilibrium constant, however, is questionable above pressures of 2 atmospheres and temperatures of 800°R. Moreover, N_2O_4, which comprised most of the mixture throughout the collapse, does not obey the ideal gas law very well at the high temperatures and pressures mentioned above.

Unfortunately, however, very little N_2O_4 ever dissociated, so a significant shift in the equilibrium was not achieved.

The calculated results just discussed in which the equilibrium reaction between NO_2 and N_2O_4 could not be effectively utilized to arrest the implosion, may be typical of all equilibrium reactions. Nevertheless, these tentative results should not deter future investigation in this direction.

The remainder of the discussion is devoted to the possible effects of immersing the spheres in liquids other than water.

The three parameters ρ_∞, B, and n appearing in the equation

$$\left(\frac{P+B}{P_\infty+B}\right) = \left(\frac{\rho}{\rho_\infty}\right)^n \qquad [1.4]$$

specify the liquid which implodes upon the gas. Theoretically, the isentropic sound speed in the undisturbed liquid can be determined from Equation [1.4], i.e.,

$$c_\infty = \left(\left(\sqrt{\frac{\partial p}{\partial \rho}}\right)_s\right)_{\substack{p=p_\infty \\ \rho=\rho_\infty}} = \sqrt{\frac{Bn}{\rho_\infty}}$$

as a function of ρ_∞, B, and n so that c_∞ does not independently specify the liquid. The parameter ρ_∞ influences only the period of collapse, not the pressure. This can be readily verified by substituting $\lambda\rho_\infty$, where λ = constant, and $\sqrt{\lambda}\,t$ for ρ_∞ and t respectively in Equations [1.1], [1.3]. Noting that c_∞ becomes $c_\infty/\sqrt{\lambda}$, the substitutions leave Equation [1.1] unchanged. The influence which B and n have on the peak collapse pressure are summarized in Figure 3 in which the peak collapse pressures are plotted as functions of water depth (i.e., ambient pressure determined in every case by multiplying depth by the density of water). The influences of B and n are not nearly as pronounced as those of γ and P_0 (Figures 1 and 2), the specific heat ratio and initial internal pressure of the gas inside the sphere, respectively. It appears that decreases in the values of B and n result in decreased peak internal bubble pressures.

A rough explanation, similar to that made for gases, can be made for the behavior summarized in Figure 3. It is again based on adiabatic compressibility, this time for the liquid. The adiabatic compressibility, defined as

$$\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s$$

can be immediately calculated from Equation [1.4] with the help of the relationship

$$\frac{\rho}{\rho_\infty} = \frac{v_\infty}{v}$$

Since Equation [1.4] has already been evaluated for an adiabatic process, the result is

$$\frac{1}{v}\frac{dv}{dP} = \frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s = \frac{1}{n(P+B)}$$

Thus the compressibility of the liquid is increased by a decrease in P (or p_∞), B, and n. In terms of compressibility, the behavior of the liquid is opposite to that of the gas; an increase in the compressibility of the liquid is associated with a decrease in peak collapse pressure.

In order to illustrate the above, a comparison may be made between two liquids of equal density, one of which is considerably more compressible than the other. Shortly after the beginning of the collapse the velocity of the liquid at the bubble wall for a compressible liquid is about the same as that for an incompressible liquid. Further away from the wall, however, the particle velocity of a compressible liquid will be less than that of an incompressible liquid. Moreover, at a given time a greater volume of liquid will be in motion if the liquid is incompressible. In fact, the liquid at a distance of about $c_\infty t$ from the bubble center (t being time beginning at the instant of collapse) will be in motion; the incompressible liquid has a larger value of c_∞ than does the compressible liquid. As a result, the total momentum of the inrushing liquid will be higher for incompressible liquids than for compressible liquids at the same instant in time. As time increases, the velocity of the incompressible liquid becomes increasingly greater than that of the compressible liquid at the same distance from the center of the bubble. In addition, increasingly more liquid is set into motion. The result is that the total momentum of the inrushing liquid is increasingly greater for the incompressible liquid up to bubble minimum. The difference in the final momenta of the two liquids at the bubble wall influences the difference in peak internal bubble pressures.

Keeping in mind the sympathetic implosion problem mentioned earlier, the behavior described above suggests surrounding buoyancy spheres with a very compressible substance. Use of such a substance, however, might lead to a large decrease in buoyancy at great depths.

Equation [1.4] is a modification of the Tait equation of state for a liquid undergoing an isentropic process. The writer was unable to find the parameters B and n tabulated for any liquids other than water. However, it may be possible to obtain good estimates of their values for several hydraulic liquids by fitting Equation [1.4] to the compressibility data of Hayward.[16,17]

SUMMARY AND CONCLUSIONS

1. A complete set of equations has been derived for the isentropic behavior of a Beattie-Bridgeman gas.

2. A Fortran IV computer program has been coded (see Appendix C) for the IBM 7090 digital computer to determine the behavior of a collapsing gas bubble in liquid when the gas obeys the ideal gas law.

3. A Fortran IV computer program has been coded (see Appendix B) for the IBM 7090 digital computer to determine the behavior of a collapsing gas bubble in liquid when the gas obeys the Beattie-Bridgeman equation of state.

4. Results of Items 2 and/or 3 indicate that:

a. the ideal gas law provides a reasonably accurate description of the gas inside a collapsing bubble.

b. increasing the initial internal pressure of the gas inside the bubble effectively decreases the peak collapse pressure.

c. increasing the value of γ of the gas inside the bubble effectively decreases the peak collapse pressure.

d. decreasing the values of B and n in the equation for isentropic compression of the liquid (Equation 1.4) decreases the peak collapse pressure somewhat. Figures 12 and 13 demonstrate the extent to which the peak collapse pressure can be reduced simply by filling the spheres with argon at 10 atmospheres instead of air at 1 atmosphere. Detailed comparisons are made at water depths of 1000 and 10,000 feet.

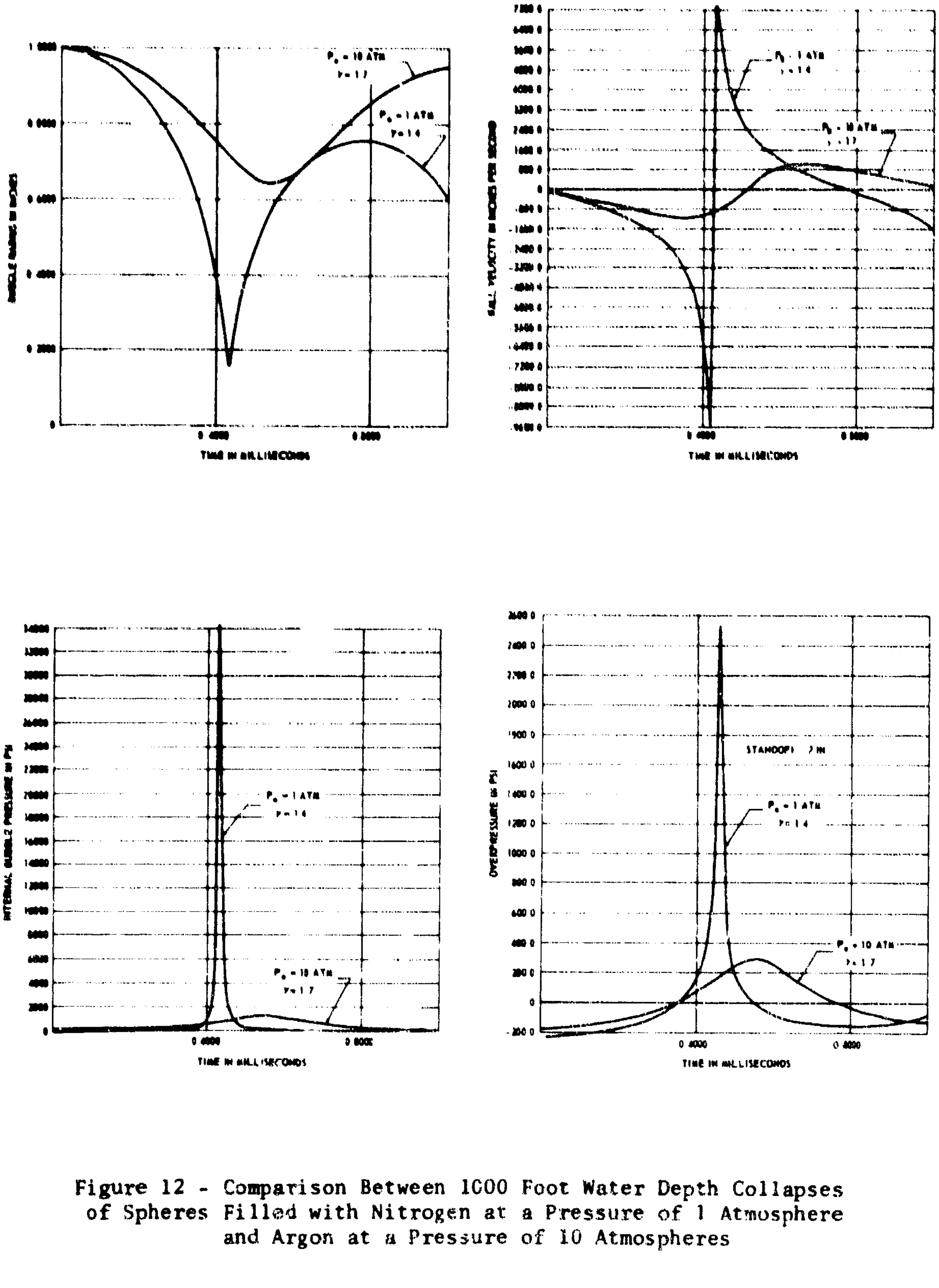

Figure 12 - Comparison Between 1000 Foot Water Depth Collapses of Spheres Filled with Nitrogen at a Pressure of 1 Atmosphere and Argon at a Pressure of 10 Atmospheres

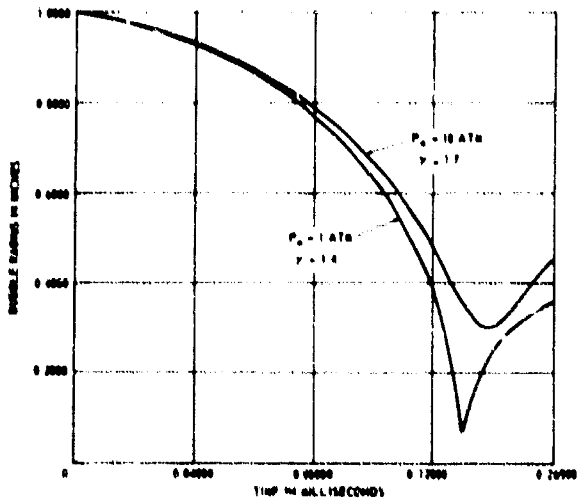

Figure 13a

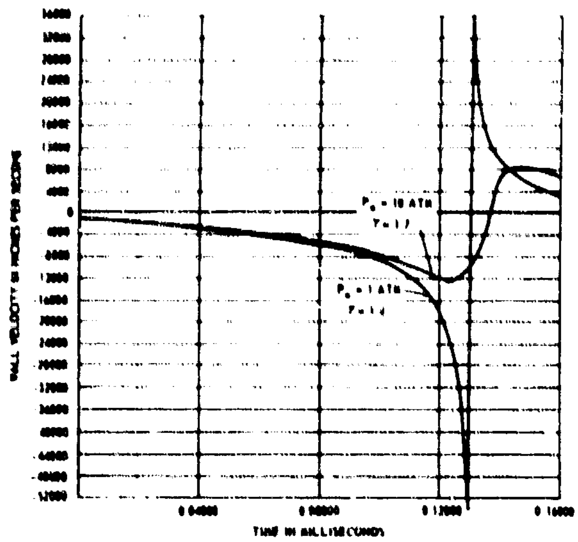

Figure 13b

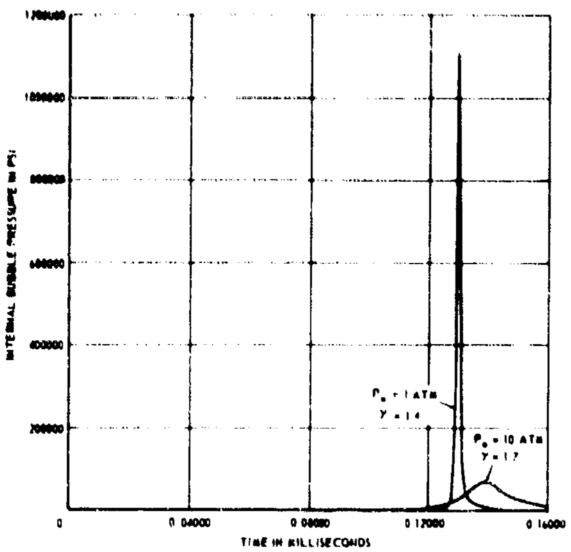

Figure 13c

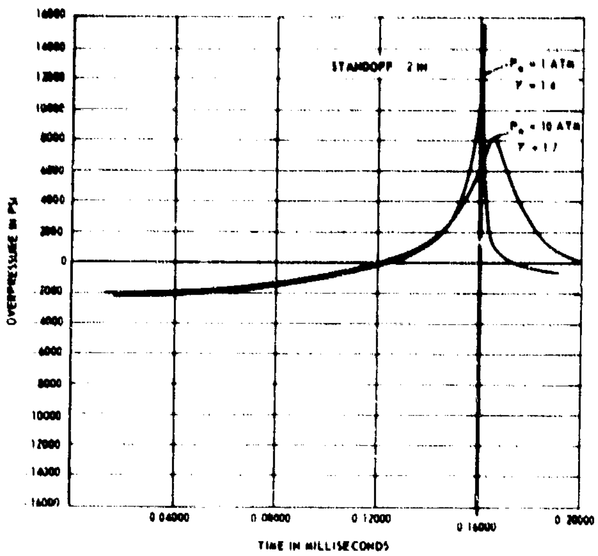

Figure 13d

Figure 13 - Comparison Between 10,000 Foot Water Depth Collapses of Spheres Filled with Nitrogen at a Pressure of 1 Atmosphere and Argon at a Pressure of 10 Atmospheres

5. The results listed in 4 verify that the peak pressure associated with a gas bubble collapse in liquid can be decreased by decreasing the adiabatic compressibility of the gas inside the bubble and/or increasing the adiabatic compressibility of the liquid in which the bubble is immersed.

6. Tentative calculations indicate that chemical reactions might be utilized to achieve the effect described in Item 5.

ACKNOWLEDGMENTS

Appreciation is expressed to Mr. S. Zilliacus for his helpful comments during the final preparation of this paper. The writer also wishes to acknowledge the efforts of Messrs. P. Shorrow, G. Hall, and I. Rappaport in preparing the figures and tables.

APPENDIX A

DETERMINATION OF AN EXPRESSION FOR $c_p = c_p(v,T)$ FOR A GAS WHICH OBEYS THE BEATTIE-BRIDGEMAN EQUATION OF STATE

To perform the derivation in an orderly manner it is first necessary to determine three fundamental relations between c_p and c_v and P, v, and T. These relations and their derivatives can be found in the references.[11,14]

Relationship 1: For any gas,

$$\left(\frac{\partial c_v}{\partial v}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_v$$

which can be derived as follows. The definition of c_v is

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v$$

By differentiating this equation with respect to v and holding T constant there results

$$\left(\frac{\partial c_v}{\partial v}\right)_T = T\left(\frac{\partial}{\partial v}\left[\left(\frac{\partial s}{\partial T}\right)_v\right]\right)_T$$

The variable s is assumed to be at least a class II function (continuous with continuous derivatives up to and including second order) with respect to the variables T and v so that the order of differentiation may be interchanged.

$$\left(\frac{\partial c_v}{\partial v}\right)_T = T\left(\frac{\partial}{\partial T}\left[\left(\frac{\partial s}{\partial v}\right)_T\right]\right)_v$$

Using the well-known Maxwell relation

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$$

in the previous expression yields

$$\left(\frac{\partial c_v}{\partial v}\right)_T = T\left(\frac{\partial}{\partial T}\left[\left(\frac{\partial P}{\partial T}\right)_v\right]\right)_v = T\left(\frac{\partial^2 P}{\partial T^2}\right)_v$$

Relationship 2: For any gas,

$$c_p - c_v = T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial P}{\partial T}\right)_v$$

as demonstrated below. Assume that

$$s = s\ (T,v)$$

By the chain rule

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$$

Dividing by dT and holding P constant gives

$$\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial s}{\partial T}\right)_v + \left(\frac{\partial s}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$$

Multiplying through by T gives

$$T\left(\frac{\partial s}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_v + T\left(\frac{\partial s}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$$

This equation can be rewritten as

$$c_p = c_v + T\left(\frac{\partial s}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$$

by virtue of the definitions

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p$$

$$c_v = T\left(\frac{\partial s}{\partial T}\right)_v$$

Finally, use of the Maxwell relation

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$$

gives

$$c_p - c_v = T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial P}{\partial T}\right)_v$$

Relationship 3: For an ideal gas

$$c_p^o - c_v^o = \bar{R}$$

This can easily be shown by employing Relationship 2 and the equation of state for an ideal gas

$$Pv = \bar{R}T$$

Substitution of the value

$$T = \frac{Pv}{\bar{R}}$$

and the derivatives

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{\bar{R}}{v}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{\bar{R}}{P}$$

into Relationship 2 yields

$$c_p^o - c_v^o = \left(\frac{Pv}{\bar{R}}\right)\left(\frac{\bar{R}}{v}\right)\left(\frac{\bar{R}}{P}\right) = \bar{R}$$

The superscript o indicates that the variables refer to the ideal state.

Now an explicit expression for $\left(\frac{\partial c_v}{\partial v}\right)_T$ can be determined by substitution of the Beattie-Bridgeman equation of state [1.8] into Relationship 1, yielding

$$\left(\frac{\partial c_v}{\partial v}\right)_T = \frac{T\beta''}{v^2} + \frac{T\gamma''}{v^3} + \frac{T\delta''}{v^4}$$

where

$$\beta'' = -\frac{6c\bar{R}}{T^4}$$

$$\gamma'' = -\frac{6cB_o\bar{R}}{T^4}$$

$$\delta'' = \frac{6bcB_o\bar{R}}{T^4}$$

This equation can be integrated from $v = \infty$ ($P = 0$) to $v = v$ along a path of constant temperature (note that β'', γ'', and δ'' are all functions of temperature only).

$$\int_{v=\infty}^{v=v} \left(\frac{\partial c_v}{\partial v}\right)_T dv = [c_v(v,T)]_{v=v} - [c_v(v,T)]_{v=\infty}$$

$$= \left[-\frac{T\beta''}{v} - \frac{T\gamma''}{2v^2} - \frac{T\delta''}{3v^3} \right]_{v=\infty}^{v=v}$$

or

$$c_v(v,T) - [c_v(v,T)]_{v=\infty} = -T\left(\frac{\beta''}{v} + \frac{\gamma''}{2v^2} + \frac{\delta''}{3v^2}\right)$$

As the pressure approaches zero (volume approaches infinity) the properties of a real gas become less distinguishable from those of an ideal gas in the same state; in the limit, the corresponding properties are the same. If $c_v^o(T)$ is the specific constant volume heat capacity of the ideal state, then

$$[c_v(v,T)]_{v=\infty} = c_v^o(T) = c_v^o$$

It is understood that c_v^o, by definition of ideal state, depends on temperature exclusively. Then

$$c_v - c_v^o = -T\left(\frac{\beta''}{v} + \frac{\gamma''}{2v^2} + \frac{\delta''}{3v^3}\right)$$

Substitution into Relationship 3,

$$c_v^o = c_p^o - \bar{R}$$

gives

$$c_v = c_p^o - \bar{R} - T\left(\frac{\beta''}{v} + \frac{\gamma''}{2v^2} + \frac{\delta''}{3v^3}\right)$$

This expression for c_v can be substituted into Relationship 2 after Relationship 2 has been evaluated for a gas obeying the Beattie-Bridgeman equation. The final expression is the one which appears in the text.

APPENDIX B

COMPUTER PROGRAM BASED ON THE BEATTIE-BRIDGEMAN MODEL

The Fortran IV computer program RHO3 has been coded to determine numerically the behavior of a collapsing gas filled cavity in liquid when the gas obeys the Beattie-Bridgeman equation of state. The program is listed on the following pages. A Fortran IV or a binary deck can be obtained from the NSRDC Applied Mathematics Laboratory. A Fortran IV deck to be used on any IBM 7090 digital computer can be punched from the listing provided the plotting routine gplφT[18] is eliminated by following the instructions on four of the comment cards. Note that if these instructions are followed, the subprogram GPLφT and SPACES can be excluded.

```
$IBSYS
$ATTACH        B9
$AS            SYSLB4
$EXECUTE       IBJOB
$IBJOB         MAP,FIOCS
$IBFTC RUN1    NODECK,SDD
C PROGRAMMER  R. LILLISTON   CODE 745   EXT 3291
C  H IN FEET, RC IN INCHES, PO IN LBS/SQ IN, TEMPO IN DEG RANKINE.
C  PCRIT IN ATMOS, TCRIT IN DEG RANKINE
      REAL MCCV,MGCR,MUDU,MODP
      DIMENSION XX(1001),YY(1001,5),ACC(1000),TEMP(1000),ADCOMP(1000),TI
     1TLEX(5),TITLEV(5),BTITLE(8),Z(1000)
C  IF THIS PROGRAM IS BEING REPRODUCED AND USED ON A COMPUTER FACILITY
C  OTHER THAN THAT AT N.S.R.D.C. ELIMINATE THE NEXT CARD
      COMMON XX,YY,BTITLE,TITLEV,TITLEX
      COMMON/AT/RG,PL,B,DL,EN,C,CPHCA,CPHCB,CPHCC,CPHCD,BBA,BBB,BBC,BBAO
     1,BBBO
C VALUES OF THE CONSTANTS FOR THE LIQUID - WATER
      B=4.4106
      C=0.084
      EN=7.0
      DL=2.0
      PA=14.7
      RG=2.71123
      DO 2 K=1,50
      READ(5,7)H,RO,PO,TEMPO,BBAO,BBBO,BBA,BBB,BBC,CPHCA,CPHCB,CPHCC,CPH
     1CD,XNAME,YNAME
    7 FORMAT(4F10.4/E15.4,F10.4,3E15.4,2A5)
      IF(RO)55,999,55
   55 CONTINUE
      PL=DL*H*32.2/144.0+PA
      WRITE(6,11)RO,H,PO,XNAME,YNAME,TEMPO
   11 FORMAT(1H1/////////40X,44H GILMORES SECOND ORDER APPROXIMATION FOR
     1 THE///44X,14HIMPLOSION OF A,F7.2,19H INCH RADIUS SPHERE///45X,14H
     2 AT A DEPTH OF,F8.1,14H FEET OF WATER///31X,51HWHEN THE INITIAL IN
     3TERNAL PRESSURE IN THE SPHERE IS,F9.4,5H PSIA///40X,21HAND THE GAS
     4 INSIDE IS,1X,2A5///40X,28HAT AN INITIAL TEMPERATURE OF,F5.0,16H D
     5EGREES RANKINE)
      WRITE(6,12)
   12 FORMAT(1H1/10X,4HTIME,8X,6HRADIUS,7X,8HVELOCITY,5X,12HACCELERATION
     1,5X,8HPRESSURE,6X,11HTEMPERATURE,1EH COMPRESSIBILITY,2X,10H MIGRAT
     2ION/5X,14H(MILLISECONDS),2X,8H(INCHES),6X,8H(IN/SEC),5X,12H(IN/SEC
     3/SEC),4X,10H(LBS/SQIN),7X,7H(DEG.R),6X,9H(SQIN/LB),5X,9H (INCHES)/
     4//)
C  CALC VO USING NEWTON TYPE PROCEDURE FOR FINDING ROOTS OF POLYNOMIALS
      VO=14.7*RG*TEMPO/PO
      DO 27 J=1,100
      FVO=PO*VO**4/14.7-RG*TEMPO*VO**3+(BBAO+RG*BBC/TEMPO**2-RG*TEMPO*BB
     1BO)*VO**2+(RG*TEMPO*BBB*BBBO+RG*BBC*BBBO/TEMPO**2-BBA*BBAO)*VO-RG*
     2BBC*BBBO*BBBO/TEMPO**2
      DFVO=4.0*PO*VO**3/14.7-3.0*RG*TEMPO*VO**2+2.0*(BBAO+RG*BBC/TEMPO**
     12-RG*TEMPO*BBBO)*VO+(RG*TEMPO*BBB*BBBO+RG*BBC*BBBO/TEMPO**2-BBA*BB
     2AO)
      TVO=VO-FVO/DFVO
      IF(J-99)205,200,200
  205 IF(ABS(VO-TVO)-VO*1.0E-4)201,202,202
  202 VO=TVO
      GO TO 27
  200 WRITE(6,211)
  211 FORMAT(1H1///5X,35H ITERATION FOR VO DOES NOT CONVERGE)
      GO TO 999
```

```
   27 CONTINUE
  201 VO=TVC
      S=4.0*(1.0E-5)*RO*SQRT(DL/(PL-PO))
C     CALCULATION OF THE COEFFICIENTS OF THE TAYLOR SERIES FOR THE INITIAL
C       VALUES OF THE RADIUS
      DGO=1.0/VO
      R2=-(2.073OE4)*EN*(PL+B)*(1.0-((PO+B)/(PL+B))**((EN-1.0)/EN))/((EN
     1-1.0)*DL*2.0*RO)
      CPNUM=1.0/DGO**3+(BBBO+2.0*BBC/TEMPO**3)/DGO**2+BBBO*(2.0*BBC/TEMP
     1O**3-BBB)/DGO-2.0*BBC*BBB*BBBO/TEMPO**3
      CPDEN=RG*TEMPO/DGO**3+2.0*(BBBO*RG*TEMPO-BBAO-BBC*RG/TEMPO**2)/DGO
     1**2+3.0*(BBA*BBAO-BBB*BBBO*RG*TEMPO-BBC*BBBO*RG/TEMPO**2)/DGO+4.0*
     2BBC*BBB*BBBO*RG/TEMPO**2
      CPTO=0.168*(CPHCA+CPHCB*5.0*TEMPO/9.0+CPHCC*(5.0*TEMPO/9.0)**2+CPH
     1CD*(5.0*TEMPO/9.0)**3)-RG+6.0*BBC*RG*DGO/TEMPO**3*(1.0+BBBO*DGO/2.
     20-BBBC*BBB*DGO**2/3.0)+RG**2*TEMPO*DGO*(BBBO+1.0/DGO-BBB*BBBO*DGO)
     3*(1.0+2.0*BBC*DGO/TEMPO**3)*CPNUM/CPDEN
      PNUM=2.0*PC/(14.7*DGO)-RG*TEMPO-RG*BBBO*BBC*DGO**2/TEMPO**2+2.0*RG
     1*BBBO*BBB*BBC*DGO**3/TEMPO**2-RG*TEMPO*BBBO*BBB*DGO**2+BBAO*BBA*DG
     2O**2
      PDEN=-CPTO/DGO+RG/DGO+2.0*RG*BBC/TEMPO**3+RG*BBBO-RG*BBBO*BBB*DGO+
     12.0*RG*BBBC*BBC*DGO/TEMPO**3-2.0*RG*BBBO*BBC*BBB*DGO**2/TEMPO**3
      P2=44.1*R2*PNUM/(RO*PDEN)
      R3=2.0*R2**2*((PL+B)/(PO+B))**((EN-1.0)/(2.0*EN))/(3.0*C)
     1-(2.073OE4)*EN*(PL+B)*R2/(3.0*C*(EN-1.0)*DL*RO)*(((PL+B)/(PO+B))**
     2((EN-1.0)/(2.0*EN))-((PO+B)/(PL+B))**((EN-1.0)/(2.0*EN)))+
     3(2.073OE4)*P2/(3.0*C*DL)*((PL+B)/(PO+B))**((EN+1.0)/(2.0*EN))
      D2ALFA=(-BBA*BBAO+BBAO/DGO+PO/(14.7*DGO**3))*(6.0*R2/(DGO**2*RO)+
     13.0*BBBO*R2/(DGO*RO))/(RG*(1.0/DGO**2+BBBO/DGO-BBBO*BBB)**2)-(3.0*
     2BBAO*R2/(DGO*RO)+P2/(14.7*DGO**3)+9.0*PO*R2/(14.7*RO*DGO**3))/(RG*
     3(1.0/DGO**2+BBBO/DGO-BBB*BBBO))
      D2BETA=1.0*BBC*DGO*R2/RO
      ALFAO=(BBA*BBAO-BBAO/DGO-PO/(14.7*DGO**3))/(RG*(1.0/DGO**2+BBBO/DG
     1O-BBBC*BBB))
      BETAO=-BBC*DGO
      D2TEMP=-D2ALFA/3.0+(-D2BETA/2.0-D2ALFA*ALFAO**2/9.0+(BETAO*D2BETA/
     12.0 +ALFAO**3*D2BETA/27.0+BETAO*ALFAO**2*D2ALFA/9.0)/(2.0*SQRT(BET
     2AO**2/4.0+BETAO*ALFAO**3/27.0)))/(3.0*(-BETAO/2.0-ALFAO**3/27.0+SQ
     3RT(BETAO**2/4.0+BETAO*ALFAO**3/27.0))**(2.0/3.0))+(-D2BETA/2.0-ALF
     4AO**2*D2ALFA/9.0-(BETAO*D2BETA/2.0+ALFAO**3*D2BETA/27.0+ALFAO**2*B
     5ETAO*D2ALFA/9.0)/(2.0*SQRT(BETAO**2/4.0+BETAO*ALFAO**3/27.0)))/(
     63.0*(-BETAO/2.0-ALFAO**3/27.0-SQRT(BETAO**2/4.0+BETAO*ALFAO**3/27.
     70))**(2.0/3.0))

      D2CNUM=(9.0/(DGO**3*RO)+6.0/(DGO**2*RO)*(BBBO+2.0*BBC/TEMPO**3)+3.
     10*BBBO*(-BBB+2.0*BBC/TEMPO**3)/(DGO*RO))*(2.0*R2)+6.0*BBC/TEMPO**4
     2*(BBB*BBBO-BBBO/DGO-1.0/DGO**2)*D2TEMP
      D2CDEN=(9.0*RG*TEMPO/(DGO**3*RO)+12.0/(DGO**2*RO)*(-BBAO+BBBO*RG*T
     1EMPO-BBC*RG/TEMPO**2)+9.0/(DGO*RO)*(BBA*BBAO-BBB*BBBO*RG*TEMPO-BBB
     2O*BBC*RG/TEMPO**2))*(2.0*R2)+(RG/DGO**3+2.0*RG/DGO**2*(BBBO+2.0*BB
     3C/TEMPO**3)-3.0*BBBO*RG/DGO*(BBB-2.0*BBC/TEMPO**3)-8.0*BBB*BBC*BBB
     4O*RG/TEMPO**3)*D2TEMP
      D2CPTO=0.368*(5.0*CPHCB/9.0+2.0*TEMPO*CPHCC*(5.0/9.0)**2+3.0*CPHCD
     1*TEMPO**2*(5.0/9.0)**3)*D2TEMP-18.0*BBC*RG*DGO/TEMPO**4*(1.0+BBBO*
     2DGO/2.0-BBBO*BBB*DGO**2/3.0)*D2TEMP-18.0*BBC*RG*DGO/(TEMPO**3*RO)*
     3(1.0+BBBC*DGO/2.0-BBBO*BBB*DGO**2/3.0)*2.0*R2+3.0*BBBO*BBC*RG*DGO*
```

```
     4**2/(RO*TEMPO**3)*(4.0*DGO*BBB-3.0)*2.0*R2+RG**2*DGO*(BBBO+1.0/DGO-
     5BBB*BBB*DGO)*(1.0+2.0*BBC*DGO/TEMPO**3)*CPNUM/CPDEN*D2TEMP-3.0*RG
     6**2*TEMPO*DGO/RO*(BBBO+1.0/DGO-BBB*BBBO*DGO)*(1.0+2.0*DGO*BBC/TEMP
     7O**3)*CPNUM/CPDEN*2.0*R2+3.0*RG**2*TEMPO*DGO/RO*(1.0/DGO+BBO*BBBO*
     8DGO)*(1.0+2.0*BBC*DGO/TEMPO**3)*CPNUM/CPDEN*2.0*R2-6.0*BBC*RG**2*D
     9GO**2/TEMPO**2*(BBBO+1.0/DGO-BBB*BBBO*DGO)*(D2TEMP/TEMPO+2.0*R2/RO
     1)*CPNUM/CPDEN+RG**2*TEMPO*DGO*(BBBO+1.0/DGO-BBB*BBBO*DGO)*(1.0+2.0
     2*BBC*DGO/TEMPO**3)*(D2CNUM/CPDEN-CPNUM*D2CDEN/CPDEN**2)
      D2PNUM=4.0/(14.7*DGO)**2+(6.0*PO/(14.7*DGO*RO)+6.0*RG*BBBO*BBC*DGO
     1**2/(TEMPO**2*RO)-18.0*RG*BBBO*BBB*BBC*DGO**3/(TEMPO**2*RO)+6.0*RG
     2*TEMPO*BBC*BBB*DGO**2/RO-6.0*BBAO*BBA*DGO**2/RO)*(2.0*R2)+(2.0*RG
     3*BBBO*BBC*DGO**2/TEMPO**3-RG-4.0*RG*BBBO*BBB*BBC*DGO**3/TEMPO**3
     4-RG*BBBO*BBB*DGO**2)*D2TEMP
      D2PDEN=-D2CPTO/DGO+(3.0*RG/(DGO*RO)-3.0*CPTO/(DGO*RO)+3.0*RG*BBBO*
     1BBB*DGO/RO-6.0*RG*BBBO*BBC*DGO/(RO*TEMPO**3)+12.0*RG*BBBO*BBB*BBC*
     2DGO**2/(RO*TEMPO**3))*2.0*R2+6.0*RG*BBC/TEMPO**4*(BBBO*BBB*DGO**2-
     3BBBO*DGO-1.0)*D2TEMP
      P3=44.1/(6.0*RO)*(CPTO*6.0*R3*PNUM/PDEN)
      R4=3.0*R3*R2/C*((PL+B)/(PO+B))**((EN-1.0)/(2.0*EN))-2.0*R2   /(3.0
     1*RO)-(2.0736E4)*EN*(PL+B)/((EN-1.0)*DL*RO*24.0)*(-(EN-1.0)/(EN*(PL
     2+B)**((EN-1.0)/EN)*(PO+B)**(1.0/EN))*2.0*P2+6.0*R3/C*((PL+B)/(PO+B
     3))**((EN-1.0)/(2.0*EN)))+(2.0736E4)/(C*DL*4.0)*P3*((PL+B)/(PO+B))*
     4*((EN+1.  )/(2.0*EN))-(2.0736E4)*P2*R2/(C**2*DL)*(PL+B)/((PO+B)*3.)
      P4=-44.1/24.0*CPTO/RO**2*(PNUM/PDEN)*4.0*R2**2+44.1*PNUM/(RO*PDEN*
     124.0)*(CPTO*24.  *R4+6.0*R2*D2CPTO-8.0*CPTO*R2**2/RO)+132.3/(12.0*R
     2O)*CPTO*R2*(1.0/PDEN*D2PNUM-PNUM*D2PDEN/PDEN**2)
C   CALCULATION OF FOUR INITIAL VALUES
      XX(1)=0.0
      RNM3=RO
      PNM3=PO
      UNM3=0.0
      XX(2)=S
      RNM2=RO+R2*XX(2)**2+XX(2)*R3*XX(2)**2+XX(2)**2*R4*XX(2)**2
      PNM2=PO+P2*XX(2)**2+XX(2)*P3*XX(2)**2+XX(2)**2*P4*XX(2)**2
      UNM2=2.0*R2*XX(2)+3.0*R3*XX(2)**2+4.0*XX(2)*R4*XX(2)**2
      XX(3)=2.0*S
      RNM1=RO+R2*XX(3)**2+XX(3)*R3*XX(3)**2+XX(3)**2*R4*XX(3)**2
      PNM1=PO+P2*XX(3)**2+XX(3)*P3*XX(3)**2+XX(3)**2*P4*XX(3)**2
      UNM1=2.0*R2*XX(3)+3.0*R3*XX(3)**2+4.0*XX(3)*R4*XX(3)**2
      XX(4)=3.  *S
      RN=RO+R2*XX(4)**2+XX(4)*R3*XX(4)**2+XX(4)**2*R4*XX(4)**2
      PN=PO+P2*XX(4)**2+XX(4)*P3*XX(4)**2+XX(4)**2*P4*XX(4)**2
      UN=2.0*R2*XX(4)+3.0*R3*XX(4)**2+4.0*XX(4)*R4*XX(4)**2
      KKKK=1
  752 VNM3=VO*(RNM3/RO)**3
      VNM2=VO*(RNM2/RO)**3
      VNM1=VO*(RNM1/RO)**3
      VN=VO*(RN/RO)**3
      DVNM3=3.  *VNM3*UNM3/RNM3
      DVNM2=3.0*VNM2*UNM2/RNM2
      DVNM1=3.  *VNM1*UNM1/RNM1
      DVN=3.0*VN*UN/RN
      TNM3=XTEMP(VNM3,PNM3)
      IF(TNM3)401,401,402
  401 WRITE(6,403)
  403 FORMAT(1X///43HITERATION FOR TEMPERATURE DOES NOT CONVERGE)
```

```
      GO TO 962
  402 CONTINUE
      TNM2=XTEMP(VNM2,PNM2)
      IF(TNM2)401,401,406
  406 CONTINUE
      TNM1=XTEMP(VNM1,PNM1)
      IF(TNM1)401,401,404
  404 CONTINUE
      TN=XTEMP(VN,PN)
      IF(TN)401,401,405
  405 CONTINUE
      CALL BARPRA(ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA,TNM3)
      CPBAR=XCPHC(VNM3,TNM3,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDE
     1LTA)
      DPNM3=XDPRES(VNM3,DVNM3,CPBAR,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DG
     1AMMA,DDELTA)
      DUNM3=XACCEL(RNM3,UNM3,PNM3,DPNM3)
      CALL BARPRA(ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA,TNM2)
      CPBAR=XCPHC(VNM2,TNM2,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDE
     1LTA)
      DPNM2=XDPRES(VNM2,DVNM2,CPBAR,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DG
     1AMMA,DDELTA)
      DUNM2=XACCEL(RNM2,UNM2,PNM2,DPNM2)
      CALL BARPRA(ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA,TNM1)
      CPBAR=XCPHC(VNM1,TNM1,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDE
     1LTA)
      DPNM1=XDPRES(VNM1,DVNM1,CPBAR,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DG
     1AMMA,DDELTA)
      DUNM1=XACCEL(RNM1,UNM1,PNM1,DPNM1)
      CALL BARPRA(ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA,TN)
      CPBAR=XCPHC(VN,TN,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA)
      DPN=XDPRES(VN,DVN,CPBAR,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,D
     1DELTA)
      DUN=XACCEL(RN,UN,PN,DPN)
      IF(KKKK-1)681,682,682
  682 YY(1,1)=RNM3
      YY(2,1)=RNM2
      YY(3,1)=RNM1
      YY(4,1)=RN
      YY(1,2)=UNM3
      YY(2,2)=UNM2
      YY(3,2)=UNM1
      YY(4,2)=UN
      ACC(1)=DUNM3
      ACC(2)=DUNM2
      ACC(3)=DUNM1
      ACC(4)=DUN
      YY(1,3)=PNM3
      YY(2,3)=PNM2
      YY(3,3)=PNM1
      YY(4,3)=PN
      TEMP(1)=TNM3
      TEMP(2)=TNM2
      TEMP(3)=TNM1
      TEMP(4)=TN
      ADCOMP(1)=-1.0/RNM3
```

```
      ADCOMP(2)=-UNM2/(RNM2*DPNM2)
      ADCOMP(3)=-UNM1/(RNM1*DPNM1)
      ADCOMP(4)=-UN/(RN*DPN)
      PNR=RN
      CNR=RN
      PNU=UN
      CNU=UN
      PNP=PN
      CNP=PN
      KKKK=0
      N=4
      LM=0
  681 RBAR=RNM3+4.0*S/3.0*(2.0*UN-UNM1+2.0*UNM2)
      UBAR=UNM3+4.0*S/3.0*(2.0*DUN-DUNM1+2.0*DUNM2)
      PBAR=PNM3+4.0*S/3.0*(2.0*DPN-DPNM1+2.0*DPNM2)
      KOUNT=0
  589 MODR=RBAR-112.0/121.0*(PNR-CNR)
      MODV=VO*(MODR/RO)**3
      MODU=UBAR-112.0/121.0*(PNU-CNU)
      MODP=PBAR-112.0/121.0*(PNP-CNP)
      TBAR=XTEMP(MODV,MODP)
      IF(TBAR.LE.0.0)GO TO 401
      DMODV=3. *MODV*MODU/MODR
      CALL BARCRA(ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA,TBAR)
      CPBAR=XCPHC(MODV,TBAR,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDE
     1LTA)
      DMODP=XDPRES(MODV,DMODV,CPBAR,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DG
     1AMMA,DDELTA)
      DMODU=XACCEL(MODR,MODU,MODP,DMODP)
      CORP=(9.0*PN-PNM2+3.0*S*(DMODP+2.0*DPN-DPNM1))/8.0
      CORU=(9. *UN-UNM2+3.0*S*(DMODU+2.0*DUN-DUNM1))/8.0
      CORR=(9. *RN-RNM2+3.0*S*(MODU+2.0*UN-UNM1))/8.0
      IF(ABS(PBAR-CORP)-ABS(CORP)*1.0E-3)586,586,587
  586 IF(ABS(UBAR-CORU)-ABS(CORU)*1.0E-3)588,587,587
  587 RBAR=CORR+9.0/121.0*(RBAR-CORR)
      UBAR=CORU+9.0/121.0*(UBAR-CORU)
      PBAR=CORP+9.0/121.0*(PBAR-CORP)
      KOUNT=KOUNT+1
      IF(KOUNT-500)589,589,450
  450 WRITE(6,452)
  452 FORMAT(///5X,43HTHE INTEGRATION ITERATION DOES NOT CONVERGE)
      GO TO 962
  588 XX(N+1)=XX(N)+S
      YY(N+1,1)=CORR+9.0/121.0*(RBAR-CORR)
      YY(N+1,2)=CORU+9.0/121.0*(UBAR-CORU)
      VF=VO*(YY(N+1,1)/RO)**3
      YY(N+1,3)=CORP+9.0/121.0*(PBAR-CORP)
      TEMP(N+1)=XTEMP(VF,YY(N+1,3))
      DVF=3. *VF*YY(N+1,2)/YY(N+1,1)
      CALL BARDRA(ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA,TEMP(N
     1+1))
      CPF=XCPHC(VF,TEMP(N+1),ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DD
     1ELTA)
      DPF=XDPRES(VF,DVF,CPF,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDE
     1LTA)
      ACC(N+1)=XACCEL(YY(N+1,1),YY(N+1,2),YY(N+1,3),DPF)
```

```
      ADCOMP(N+1)=-YY(N+1,2)/(YY(N+1,1)*DPF)
      N=N+1
      PNU=UHAR
      CNU=CCRU
      PNR=RDAR
      CNR=CCRR
      PNP=PEAR
      CNP=CCRP
      RNM3=RNM2
      RNM2=RNM1
      RNM1=RN
      RN=YY(N,1)
      UNM3=UNM2
      UNM2=UNM1
      UNM1=UN
      UN=YY(N,2)
      PNM3=PNM2
      PNM2=PNM1
      PNM1=PN
      PN=YY(N,3)
      DPNM3=DPNM2
      DPNM2=DPNM1
      DPNM1=DPN
      DPN=DPF
      DUNM3=DUNM2
      DUNM2=DUNM1
      DUNM1=DUN
      DUN=ACC(N)
      IF(N-990)961,961,962
  961 IF(ABS(YY(N,3)-YY(N-1,3))-.050*YY(N-1,3))681,681,606
  606 S=S/2.0
      LM=LM+1
      R4MH=(80.0*RN+135.0*RNM1+40.0*RNM2+RNM3)/256.0+S/128.0*(-15.0*UN+
     190.0*UNM1+15.0*UNM2)
      R3MH=(12.0*RN+135.0*RNM1+108.0*RNM2+RNM3)/256.0+S/128.0*(-3.0*UN
     1-54.0*UNM1+27.0*UNM2)
      U4MH=(80.0*UN+135.0*UNM1+40.0*UNM2+UNM3)/256.0+S/128.0*(-15.0*DUN
     1+90.0*DUNM1+15.0*DUNM2)
      U3MH=(12.0*UN+135.0*UNM1+108.0*UNM2+UNM3)/256.0+S/128.0*(-3.0*DUN
     1-54.0*DUNM1+27.0*DUNM2)
      P4MH=(80.0*PN+135.0*PNM1+40.0*PNM2+PNM3)/256.0+S/128.0*(-15.0*DPN
     1+90.0*DPNM1+15.0*DPNM2)
      P3MH=(12.0*PN+135.0*PNM1+108.0*PNM2+PNM3)/256.0+S/128.0*(-3.0*DPN
     1-54.0*DPNM1+27.0*DPNM2)
      RNM3=R3MH
      RNM2=RNM1
      RNM1=R4MH
      UNM3=U3MH
      UNM2=UNM1
      UNM1=U4MH
      PNM3=P3MH
      PNM2=PNM1
      PNM1=P4MH
      GO TO 7[illegible]
  962 NK=1
      S=XX(NK+1)-XX(NK)
```

```
      HP=XX(NK+2)-XX(NK+1)
      XINT1=0.
      XINT2=YY(NK,1)**3*S+((S+HP)**2*YY(NK+1,1)**3-(2.0*S*HP+HP**2)*YY(
     1NK,1)**3-S**2*YY(NK+2,1)**3)/(2.0*HP*(S+HP))*S+(S*YY(NK+2,1)**3-
     2(S+HP)*YY(NK+1,1)**3+HP*YY(NK,1)**3)/(3.0*HP*(S+HP))*S**2
      XINT3=YY(NK+1,1)**3*HP+(-HP**2*YY(NK,1)**3+(HP**2-S**2)*YY(NK+1,1)
     1**3+S**2*YY(NK+2,1)**3)/(2.0*S*(S+HP))*HP+(HP*YY(NK,1)**3-(S+HP)*
     2YY(NK+1,1)**3+S*YY(NK+2,1)**3)/(3.0*S*(S+HP))*HP**2+XINT2
      XPINT1=0.0
      XPINT2=XINT2/YY(NK+1,1)**3
      XPINT3=XINT3/YY(NK+2,1)**3
      Z(NK)=0.0
      Z(NK+1)=(XPINT1*S+((S+HP)**2*XPINT2-(2.0*S*HP+HP**2)*XPINT1-S**2*
     1XPINT3)/(2.0*HP*(S+HP))*S+(S*XPINT3-(S+HP)*XPINT2+HP*XPINT1)/(3.0*
     2HP*(S+HP))*S**2)*772.8
      Z(NK+2)=Z(NK+1)+(XPINT2*HP+(-HP**2*XPINT1+(HP**2-S**2)*XPINT2+S**2
     1*XPINT3)/(2.0*S*(S+HP))*HP+(HP*XPINT1-(S+HP)*XPINT2+S*XPINT3)/(3.0
     2*S*(S+HP))*HP**2)*772.8
      NK=2
  437 NK=NK+1
      S=HP
      HP=XX(NK+1)-XX(NK)
      XINT1=XINT2
      XINT2=XINT3
      XINT3=YY(NK,1)**3*HP+(-HP**2*YY(NK-1,1)**3+(HP**2-S**2)*YY(NK,1)**
     13+S**2*YY(NK+1,1)**3)/(2.0*S*(S+HP))*HP+(HP*YY(NK-1,1)**3-(S+HP)*
     2YY(NK,1)**3+S*YY(NK+1,1)**3)/(3.0*S*(S+HP))*HP**2+XINT2
      XPINT1=XPINT2
      XPINT2=XPINT3
      XPINT3=XINT3/YY(NK+1,1)**3
      Z(NK+1)=Z(NK)+(XPINT2*HP+(-HP**2*XPINT1+(HP**2-S**2)*XPINT2+S**2*
     1XPINT3)/(2.0*S*(S+HP))*HP+(HP*XPINT1-(S+HP)*XPINT2+S*XPINT3)/(3.0*
     2S*(S+HP))*HP**2)*772.8
      IF(NK+1.LT.999)GO TO 437
      DO 959 LL=1,N
  959 XX(LL)=XX(LL)*1.0E3
      READ(5,73)IDEN,STF,PCRIT,TCRIT,VCRIT
   73 FORMAT(I10,4F15.5)
C  IF THIS PROGRAM IS BEING REPRODUCED AND USED ON A COMPUTER FACILITY
C  OTHER THAN THAT AT N.S.R.D.C. ELIMINATE THE NEXT 3 CARDS
      GO TO (104,105,106),IDEN
  104 CONTINUE
  106 CONTINUE
      WRITE(6,70)(XX(L),YY(L,1),YY(L,2),ACC(L),YY(L,3),TEMP(L),ADCOMP(L)
     1,Z(L),L=1,N)
   70 FORMAT(/X,F10.8,F13.8,6E15.6)
C  IF THIS PROGRAM IS BEING REPRODUCED AND USED ON A COMPUTER FACILITY
C  OTHER THAN THAT AT N.S.R.D.C. ELIMINATE ALL CARDS BEGINNING WITH THE
C  NEXT UP TO, BUT NOT INCLUDING, THE CARD CONTAINING STATEMENT NO. 109
      IF(IDEN.EQ.1)GO TO 109
  105 CONTINUE
      DATA SCALE/6HLINEAR/
      XL=0.0
      CALL SPACE(XX(N),XR,DX)
      DX=2.0*DX
      Y2MAX=YY(1,2)
```

```
      Y2MIN=YY(1,2)
      Y3MAX=YY(1,3)
      ZMAX=Z(1)
      ZMIN=Z(1)
      DO 75 I=1,N
      IF(YY(I,2).LT.Y2MAX)GO TO 78
      Y2MAX=YY(I,2)
   78 IF(YY(I,2).GT.Y2MIN)GO TO 79
      Y2MIN=YY(I,2)
   79 IF(YY(I,3).LT.Y3MAX)GO TO 77
      Y3MAX=YY(I,3)
   77 IF(Z(I).LT.ZMAX)GO TO 74
      ZMAX=Z(I)
   74 IF(Z(I).GT.ZMIN)GO TO 729
      ZMIN=Z(I)
  729 V=VO*(YY(I,1)/RO)**3
      IF((ABS(V-VCRIT).LT.(VCRIT*0.05)).AND.(ABS(YY(I,3)-PCRIT).LT.(PCRI
     1T*0.05)).AND.(ABS(TCRIT-TEMP(I)).LT.(TCRIT*0.05)))WRITE(6,69)
   69 FORMAT(///5X,123H THE THERMODYNAMIC STATE OF THE GAS MAY BE TOO CL
     1OSE TO THE CRITICAL REGION FOR THE BEATTIE-BRIDGEMAN EQUATION TO B
     2E VALID   )
   75 CONTINUE
      CALL SPACE(RC,YT,DY)
      READ(5,76)BTITLE(1),BTITLE(2),BTITLE(3),BTITLE(4),BTITLE(5),BTITLE
     1(6),BTITLE(7),BTITLE(8)
   76 FORMAT(1 A6)
      READ(5,76)TITLEX(1),TITLEX(2),TITLEX(3),TITLEX(4),TITLEX(5),TITLEV
     1(1),TITLEV(2),TITLEV(3),TITLEV(4),TITLEV(5)
      CALL GPLOT(SCALF,1,N,0,XL,XR,0.0,YT,DX,DY)
      CALL SPACE(Y2MAX,YT,DYT)
      Y2MIN=-Y2MIN
      CALL SPACE(Y2MIN,YB,DYB)
      YB=-YB
      DY=AMAX1(DYB,DYT)
      READ(5,76)TITLEV(1),TITLEV(2),TITLEV(3),TITLEV(4),TITLEV(5)
      CALL GPLOT(SCALF,2,N,0,XL,XR,YB,YT,DX,DY)
      CALL SPACE(Y3MAX,YT,DY)
      READ(5,76)TITLEV(1),TITLEV(2),TITLEV(3),TITLEV(4),TITLEV(5)
      CALL GPLOT(SCALF,3,N,0,XL,XR,0.0,YT,DX,DY)
      CALL SPACE(ZMAX,YT,DY)
      READ(5,76)TITLEV(1),TITLEV(2),TITLEV(3),TITLEV(4),TITLEV(5)
      DO 791 NKL=1,N
  791 YY(NKL,4)=Z(NKL)
      CALL GPLOT(SCALF,4,N,0,XL,XR,0.0,YT,DX,DY)
      Y4MIN=0.0
      Y4MAX=0.
      Y5MIN=0.0
      Y5MAX=0.
  109 DO 99 I=1,N
      YK=YY(I,1)*YY(I,2)**2/2.0+YY(I,1)/DL*(YY(I,3)-PL)*(1.0-(YY(I,3)-PL
     1)/(2.0*DL*C**2))*2.0736E4
      XK3=C**2/YK**2*YY(I,1)**2*YY(I,2)*(1.0-YY(I,2)**2/(2.0*C**2))
     1-C**2/YK*YY(I,1)*(1.0-YY(I,2)/C)
      YY(I,4)=YK/(C*STF)+XK3*YK**2/(C**3*STF**2)*(1.0-YK/(STF*C**2)+(XK3
     1**2/(2. *C**4))*(YK**4/(STF**4*C**4)))
      YY(I,5)=CL*(YK/STF-YY(I,4)**2/2.0)/2.0736E4+DL/(2.0*C**2)*(YK/STF-
```

```
     1YY(I,4)**2/2.0)**2/2.0736E4
      XX(I)=XX(I)+(STF-YY(I,1))/C*(1.0-YY(I,2)*YY(I,1)/(C*STF))*1.0E3
      IF(YY(I,4).LT.Y4MAX)GO TO 96
      Y4MAX=YY(I,4)
   96 IF(YY(I,4).GT.Y4MIN)GO TO 97
      Y4MIN=YY(I,4)
   97 IF(YY(I,5).LT.Y5MAX)GO TO 98
      Y5MAX=YY(I,5)
   98 IF(YY(I,5).GT.Y5MIN)GO TO 99
      Y5MIN=YY(I,5)
   99 CONTINUE
C     IF THIS PROGRAM IS BEING REPRODUCED AND USED ON A COMPUTER FACILITY
C     OTHER THAN THAT AT N.S.R.D.C. ELIMINATE ALL CARDS BEGINNING WITH THE
C     NEXT UP TO, BUT NOT INCLUDING, THE CARD CONTAINING STATEMENT NO. 108
      IF(IDEN.EQ.1)GO TO 108
      CALL SPACE(XX(N),XR,DX)
      DX=2.0*DX
      CALL SPACE(Y4MAX,YT,DYT)
      Y4MIN=-Y4MIN
      CALL SPACE(Y4MIN,YB,DYB)
      YB=-YB
      DY=AMAX1(DYT,DYB)
      READ(5,76)TITLEV(1),TITLEV(2),TITLEV(3),TITLEV(4),TITLEV(5)
      CALL GPLOT(SCALE,4,N,0,XL,XR,YB,YT,DX,DY)
      CALL SPACE(Y5MAX,YT,DYT)
      Y5MIN=-Y5MIN
      CALL SPACE(Y5MIN,YB,DYB)
      YB=-YB
      DY=AMAX1(DYB,DYT)
      READ(5,76)TITLEV(1),TITLEV(2),TITLEV(3),TITLEV(4),TITLEV(5)
      CALL GPLOT(SCALE,5,N,0,XL,XR,YB,YT,DX,DY)
      IF(IDEN.EQ.2)GO TO 2
  108 WRITE(6,137)STF
  137 FORMAT(1H1////20X,41HEULERIAN VELOCITY AND OVERPRESSURE IN THE//21
     1X,24H LIQUID AT A STANDOFF OF,F7.2,7H INCHES////18X,4HTIME,9X,17HE
     2ULERIAN VELOCITY,6X,12HOVERPRESSURE/13X,14H(MILLISECONDS),3X,19H(I
     3NCHES PER SECOND),4X,15H(LBS PER SQ IN)///)
      WRITE(6,139)(XX(I),YY(I,4),YY(I,5),I=1,N)
  139 FORMAT(1 X,F10.3,8X,E12.5,8X,E12.5)
    2 CONTINUE
  999 STOP
      END


$IBFTC BARBR    NODECK,SCC
      SUBROUTINE BARBRA(ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA,
     1TEMP)
      COMMON/AB/RG,PL,B,DL,EN,C,CPHCA,CPHCB,CPHCC,CPHCD,BBA,BBB,BBC,BBAC
     1,BBBD
      ALFA=RG*TEMP
      BETA=-BBAC+BBBD*RG*TEMP-BBC*RG/TEMP**2
      GAMMA=B*BBCOAC-BBD*BBBD*RG*TEMP-BBC*BBBD*RG/TEMP**2
      DELTA=BBB*BBC*BBBD*RG/TEMP**2
      DALFA=RG
      DBETA=BBBD*RG+2.0*BBC*RG/TEMP**3
      DGAMMA=-(BB*BBDC*RG+2.0*BBC*BBBD*RG/TEMP**3
      DDELTA=-2.0*BBB*BBC*BBBD*RG/TEMP**3
      RETURN
      END
```

```
$IBFTC XTEMPT   NCDECK,SCD
      FUNCTION XTEMP(V,P)
      COMMON/SF/RG,PL,B,DL,EN,C,CPHCA,CPHCB,CPHCC,CPHCD,BOA,BOB,BBC,BBAC
     1,BBDO
      TEMP=P*V/(14.7*RG)
      DO 10 K=1,100
      FTEMP=RG*(V+BBBO*(1.0-BBB/V))*TEMP**3-(P*V**2/14.7+BBAO*(1.0-BBA/V
     1))*TEMP**2-RG*BBC/V*(V+BBBO*(1.0-BBB/V))
      DFTEMP=3.0*RG*(V+BBBO*(1.0-BBB/V))*TEMP**2-2.0*(P*V**2/14.7+BBAO*(
     11.0-BBA/V))*TEMP
      TTEMP=TEMP-FTEMP/DFTEMP
      IF(K-99)20,30,30
   30 XTEMP=0.
      GO TO 6
   20 IF(ABS(TEMP-TTEMP)-TTEMP*1.E-4)200,100,100
  100 TEMP=TTEMP
   10 CONTINUE
  200 XTEMP=TTEMP
    6 RETURN
      END


$IBFTC XCPHCC   NCDECK,SCD
      FUNCTION XCPHC(V,T,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,DDELTA
     1)
      COMMON/SF/RG,PL,B,DL,EN,C,CPHCA,CPHCB,CPHCC,CPHCD,BOA,BOB,BBC,BBAC
     1,BBDO
      XCPHC=4.1677*(CPHCA+CPHCB*5.0*T/9.0+CPHCC*(5.0*T/9.0)**2+CPHCD*(5.
     10*T/9.0)**3)+6.0*BBC*RG/T**3*(1.0/V+BBBO/(2.0*V**2)-BBB*BBBO/(3.0*
     2V**3))-RG+T*(V+BBBO-BBBO*BBB/V)*(RG/V**2+2.0*BBC*RG/(V*T)**3)*(DAL
     3FA*V**4+DBETA*V**3+DGAMMA*V**2+DDELTA*V)/(ALFA*V**3+2.0*BETA*V**2+
     43.0*GAMMA*V+4.0*DELTA)
      RETURN
      END


$IBFTC XCPRE    NCDECK,SCD
      FUNCTION XCPRES(V,DV,CP,ALFA,BETA,GAMMA,DELTA,DALFA,DBETA,DGAMMA,D
     1DELTA)
      COMMON/AF/RG,PL,B,DL,EN,C,CPHCA,CPHCB,CPHCC,CPHCD,BOA,BOB,BBC,BBAC
     1,BBDO
      T=ALFA/RG
      P=14.7/V**2*(RG*T*(V+BBBO*(1.0-BBB/V))*(1.0-BBC/(V*T**3))-BBAO*(1.
     10-BBA/V))
      TPVDT=T*(V+BBBO*(1.0-BBB/V))*(1.0+2.0*BBC/(V*T**3))/(2.0*P*V/(14.7
     1*RG)+BBAO*BBA/(RG*V**2)-T-BBBO*BBB*T/V**2-BBC*BBBO/(V*T)**2+2.0*BB
     2C*BBB*BBBO/(V**3*T**2))
      COEF1=(DALFA+DBETA/V+DGAMMA/V**2+DDELTA/V**3)*TPVDT
      COEF2=CP*V
      RES=14.7*CP*DV*(ALFA/V+2.0*BETA/V**2+3.0*GAMMA/V**3+4.0*DELTA/V**4
     1)
      XCPRES=RES/(COEF1-COEF2)
      RETURN
      END
```

```
$IBFTC XACCR    NODECK,SDD
      FUNCTION XACCRL(R,U,M,DP)
      COMMON/A1/RG,PL,H,DL,EN,C,CPHCA,CPHCB,CPHCC,CPHCD,BHA,BHB,BHC,BHAC
     1,PHID
      DENTH=(2.07306E4)*DP/DL*((PL+H)/(R+H))**(1.0/EN)
      ENTH=(2.73E4)*EN*(PL+H)/((EN-1.0)*DL)*(((H+R)/(PL+H))**((EN-1.0)
     1/EN)-1.0)
      CC=C*(((R+H)/(PL+H))**((EN-1.0)/(2.0*EN)))
      XACCRL=-(1.5*U**2*(1.0-U/(3.0*CC))/(R*(1.0-U/CC))+ENTH*(1.0+U/CC)/(
     1R*(1.0-U/CC))+DENTH/CC
      RETURN
      END
```

```
$IBLDR GPLOT                                                  06/10/66
$IBLDR APLOTV                                                 06/15/65
$IBLDR APHKTV                                                 06/15/65
$IBLDR RITERG        GPLOT binary
$IBLDR TABLIG            deck
$IBLDR VCHARV
$IBLDR HELLV
$IBLDR FLOTV
```

```
$IBFTC SPACEN   NODECK,SDD
      SUBROUTINE SPACE(ENDPT,XR,DX)
      IF(ENDPT.LE.1.0E-3)GO TO 68
      XID=ALOG10(ENDPT*10.0.0)-3.0
      IF(XID.GE.0.0)GO TO 114
      ID=XID-1.0
      GO TO 61
  114 ID=XID
      IF(ID.GE.1)GO TO 366
   61 IXR=ENDPT*10.0**(1-ID)
      XR=IXR+1
      XR=XR/10.0**(1-ID)
      IF(IXR.LE.20)GO TO 361
      DX=IXR/20
      DX=DX/10.0**(1-ID)
      GO TO 111
  361 DX=1.0/10.0**(1-ID)
      GO TO 111
  366 IXR=ENDPT/10.0**(ID-1)
      XR=IXR+1
      XR=XR*10.0**(ID-1)
      IF(IXR.LE.20)GO TO 136
      DX=IXR/20
      DX=DX*10.0**(ID-1)
      GO TO 111
  136 DX=1.0*10.0**(ID-1)
      GO TO 111
   68 WRITE(6,67)
   67 FORMAT(1H1///10X,46H THE ENDPOINT IS TOO SMALL FOR THIS SUBROUTINE
     1)
  111 RETURN
      END
```

For each bubble collapse, information must be read in on data cards in the following way:

Card 1

Cols 1-10	collapse depth in feet of water,	F10.4
Cols 11-20	initial sphere radius in inches,	F10.4
Cols 21-30	initial internal gas pressure in psi,	F10.4
Cols 31-40	initial temperature in degrees Rankine,	F10.4
Cols 41-50	the Beattie-Bridgeman constant A_o in $\frac{\text{atom ft}^3}{\text{mole}^2}$,	F10.4
Cols 51-60	the Beattie-Bridgeman constant B_o in $\frac{\text{ft}^3}{\text{mole}}$,	F10.4
Cols 61-70	the Beattie-Bridgeman constant a in $\frac{\text{ft}^3}{\text{mole}}$,	F10.4
Cols 71-80	the Beattie-Bridgeman constant b in $\frac{\text{ft}^3}{\text{mole}}$,	F10.4

Card 2

Cols 1-15	the Beattie-Bridgeman constant c in $\frac{\text{ft}^3}{\text{mole}}\ {}^\circ R^3$,	E15.4
Cols 16-25	$\bar{A}$, the first constant in the ideal constant pressure heat capacity equation,	F10.4
Cols 26-40	$\bar{B}$, the second constant in the ideal constant pressure heat capacity equation,	E15.4
Cols 41-55	$\bar{C}$, the third constant in the ideal constant pressure heat capacity equation,	E15.4
Cols 56-70	$\bar{D}$, the fourth constant in the ideal constant pressure heat capacity equation,	E15.4
Cols 71-80	the name of the gas inside the sphere.	

Card 3

Cols 1- 9	blank.
Col 10	1, 2, or 3 if plotting routine is incorporated in program; otherwise, blank. 1 for printed output only. 2 for plots only. 3 for printed and plotted output.

Cols 11-25	the standoff in inches,	F15.5
Cols 26-40	the critical pressure of the gas in psi,	F15.5
Cols 41-55	the critical temperature of the gas in degrees Rankine,	F15.5
Cols 56-70	the critical volume of the gas in $\frac{ft^3}{mole}$.	F15.5

Cards 4 through 10 contain graph labels. If no plots are desired (i.e., if the number appearing in Col 10 of Card 2 is 1) or if the program is not being used at NSRDC, then these cards must not be included in the data.

Card 4

Cols 1-48 Main graph title for all graphs.

Card 5

Cols 1-30 Horizontal graph label for all graphs (time in milliseconds).

Cols 31-60 Vertical label for radius-time curve (radius in inches).

Card 6

Cols 1-30 Vertical label for wall velocity-time curve (wall velocity in inches per second).

Card 7

Cols 1-30 Vertical label for bubble wall pressure-time curve (wall pressure in psi).

Card 8

Cols 1-30 Vertical label for migration-time curve (migration in inches).

Card 9

Cols 1-30 Vertical label for Eulerian velocity-time curve at the standoff given on Card 3 in Cols 11-25 (Eulerian velocity in in/sec).

Card 10

Cols 1-30 Vertical label for overpressure-time curve at the standoff given on Card 3 in Cols 11-25 (overpressure in psi).

Two blank cards in succession stop the computer. Each case requires no more than 4 minutes running time.

APPENDIX C

COMPUTER PROGRAM BASED ON THE IDEAL GAS LAW

The computer program RUO2 has been coded in Fortran IV to determine numerically the behavior of a collapsing gas filled cavity in liquid when the gas obeys the ideal gas law. The program is listed on the following pages. A Fortran IV or a binary deck is obtainable in the same way as a deck for RUO3; either through the NSRDC Applied Mathematics Laboratory, or by means of the listing and instruction comment cards therein. If the program, as listed, is run on the NSRDC computer facility, then no option between printed and plotted output is available. Output is always both printed and plotted because the plotted output may be in error. The numbers labelling the vertical axis can have no more than 6 digits, otherwise digits are dropped from the right hand side of the numbers. Since the pressures can be expected to exceed one million psi, it is advisable to check the printed out-put against the plots.

```
$IBFTC RU02
      DIMENSION XX(1001),YY(1001,5),TITLEX(5),TITLEY(5),BTITLE(8),TAU(10
     101),U(1001),TAU2(1001),YYY(1001),UU(1001),TITLE(25) ,TITL(5)
C  IF THIS PROGRAM IS BEING REPRODUCED AND USED ON A COMPUTER FACILITY
C  OTHER THAN THAT AT N.S.R.D.C. ELIMINATE THE NEXT CARD
      COMMON XX,YY,BTITLE,TITLEY,TITLEX
      COMMON/AC/CL,PL,DL,B,EN,G,PO,RO
C  DIMENSIONS OF DLREF ARE LBS-SEC**2/IN**4
      PA=14.7
   22 READ(5,1)H,RO,PO,G,STF,STF2,EN,B,DLREF,CREF
    1 FORMAT(6F12.5,F8.5/3E15.5)
      IF(RO.EQ.0.0)GO TO 999
      III=0
      PL=(9.645E-5)*H*32.2/144.0*(2.0736E4)+PA
      CL=CREF*((PL+B)/(PA+B))**((EN-1.0)/(2.0*EN))
      DL=DLREF*((PL+B)/(PA+B))**(1.0/EN)
      UO=(PO-PL)/(DL*CL)
      S=.020718 *SQRT(DL)*PO**(1.0/3.0)*RO/PL**(5.0/6.0)
C   INITIAL VALUES
      CO=CL*((PO+B)/(PL+B))**((EN-1.0)/(2.0*EN))
      HO=EN*(PL+B)/(DL*(EN-1.0))*(((PO+B)/(PL+B))**((EN-1.0)/EN)-1.0)
      DPO=-3.0*G*PO*(UO/RO)
      DHO=DPO/DL*((PL+B)/(PO+B))**(1.0/EN)
      DDRO=-UO**2/(2.0*RO)*(3.0*CO-UO)/(CO-UO)+HO*(CO+UO)/(RO*(CO-UO))+
     1DHO/CO
      DCO=CL*(EN-1.0)*DPO/(2.0*EN*(PL+B))*((PL+B)/(PO+B))**((EN+1.0)/2.0
     1*EN)
      DDPO=3.0*G*PO/RO*(UO**2*(3.0*G+1.0)/RO-DDRO)
      DDHO=DDPO/DL*((PL+B)/(PO+B))**(1.0/EN)-DPO**2/(EN*DL*(PL+B))*((PL+
     1B)/(PO+B))**((EN+1.0)/EN)
      DDDRO=2.0*UO*DDRO/RO*(UO-2.0*CO)/(CO-UO)+DDRO*(DDRO-DCO)/(CO-UO)+
     1UO**2/(2.0*RO)*(DDRO-3.0*DCO)/(CO-UO)+DHO/RO*(CO+UO)/(CO-UO)+HO/RO
     2*(DCO+DDRO)/(CO-UO)+DHO/CO*(DCO-DDRO)/(CO-UO)+((UO*DHO+RO*DDHO)*CO
     3-RO*DHO*DCO)/(RO*CO**2)
      DDCO=-CL*(EN-1.0)*(EN+1.0)*DPO**2/(2.0*EN*(PL+B))**2*((PL+B)/(PO+B
     1))**((3.0*EN+1.0)/(2.0*EN))+CL*(EN-1.0)*DDPO/(2.0*EN*(PL+B))*((PL+
     2B)/(PO+B))**((EN+1.0)/(2.0*EN))
      DDDPO=3.0*G*PO/RO*(3.0*(3.0*G+1.0)*UO*DDRO/RO-(3.0*G+2.0)*(3.0*G+
     11.0)*UO**3/RO**2-DDDRO)
      DDDHO=DDDPO/DL*((PL+B)/(PO+B))**(1.0/EN)-3.0*DPO*DDPO/(EN*DL*(PL+B
     1))*((PL+B)/(PO+B))**((EN+1.0)/EN)+DPO**3*(EN+1.0)/(EN**2*DL*(PL+B)
     2**2)*((PL+B)/(PO+B))**((2.0*EN+1.0)/EN)
      DDDDRO=(6.0*UO*DDRO**2-4.0*DDRO**2*CO-5.0*UO*DDDRO*CO-8.0*UO*DDRO*
     1DCO)/(RO*(CO-UO))+(3.0*DDRO*DDDRO-DDRO*DDCO-2.0*DDDRO*DCO)/(CO-UO)
     2+(3.5*UO**2*DDDRO-1.5*UO**2*DDCO+DDHO*CO+2.0*DHO*DCO+HO*DDCO)/(RO*
     3(CO-UO))+(HO*DDDRO+3.0*DDRO*DHO+3.0*UO*DDHO+RO*DDDHO)/(RO*(CO-UO))
     4-(3.0*UO*DDRO*DHO+2.0*UO**2*DDHO+2.0*RO*DDRO*DDHO+RO*UO*DDDHO+RO*D
     5DDRO*DHO)/(RO*CO*(CO-UO))+(2.0*UO**2*DHO*DCO+2.0*RO*DDRO*DHO*DCO+
     62.0*RO*UO*DDHO*DCO+RO*UO*DHO*DDCO)/(RO*CO**2*(CO-UO))-2.0*UO*DHO*D
     7CO**2/(CO**3*(CO-UO))
      DDDCO=CL*(EN-1.0)*(EN+1.0)*(3.0*EN+1.0)*DPO**3/(2.0*EN*(PL+B))**3*
     1((PL+B)/(PO+B))**((5.0*EN+1.0)/(2.0*EN))-3.0*CL*(EN-1.0)*(EN+1.0)*
     2DPO*DDPO/(2.0*EN*(PL+B))**2*((PL+B)/(PO+B))**((3.0*EN+1.0)/(2.0*EN
     3))+CL*(EN-1.0)*DDDPO/(2.0*EN*(PL+B))*((PL+B)/(PO+B))**((EN+1.0)/(2
     4.0*EN))
      DDDDPO=3.0*G*PO/RO*(-6.0*(3.0*G+1.0)*(3.0*G+2.0)*DDRO*UO**2/RO**2+
     13.0*(3.0*G+1.0)*DDRO**2/RO-DDDDRO+3.0*(3.0*G+1.0)*(3.0*G+2.0)*(G+1
     2.0)*UO**4/RO**3+4.0*(3.0*G+1.0)*UO*DDDRO/RO)
      DDDDHO=DDDDPO/DL*((PL+B)/(PO+B))**(1.0/EN)-(4.0*DDDPO*DPO+3.0*DDPO
     1**2)/(EN*DL*(PL+B))*((PL+B)/(PO+B))**((EN+1.0)/EN)+6.0*(EN+1.0)*DP
```

```
     20**2*DDPO/(EN**2*DL*(PL+B)**2)*((PL+B)/(PO+B))**((2.0*EN+1.0)/EN)-
     3DPO**4*(EN+1.0)*(2.0*EN+1.0)/(EN**3*DL*(PL+B)**3)*((PL+B)/(PO+B))*
     4*((3.0*EN+1.0)/EN)
      DDDDUO=(-13.0*DDRO*DDDRO*CO-12.0*DDRO**2*DCO-6.0*UO*DDDDRO*CO-15.0
     1*UO*DDDRO*DCO-12.0*UO*DDRO*DDCO-3.0*RO*DDDDRO*DCO-3.0*RO*DDDRO*DDC
     20+RO*DDRO*DDDCO+3.0*RO*DDDRO**2+4.0*RO*DDRO*DDDDRO-1.5*UO**2*DDDCO
     3+6.0*DDRO**3+22.0*UO*DDRO*DDDRO+4.5*UO**2*DDDDRO+DDDHO*CO+3.0*DDHO
     4*DCO+3.0*DHO*DDCO+HO*DDDCO+HO*DDDDRO+4.0*DDDRO*DHO+6.0*DDRO*DDHO+4
     5.0*UO*DDDHO-3.0*DDRO**2*DHO/CO-4.0*UO*DDDRO*DHO/CO+9.0*UO*DDRO*DHO
     6*DCO/CO**2+RO*DDDDHO-9.0*UO*DDRO*DDHO/CO-3.0*UO**2*DDDHO/CO+6.0*UO
     7**2*DDHO*DCO/CO**2-3.0*RO*DDDRO*DDHO/CO-3.0*RO*DDRO*DDDHO/CO+6.0*R
     8O*DDRO*DDHO*DCO/CO**2-RO*UO*DDDDHO/CO+3.0*RO*UO*DDDHO*DCO/CO**2+3.
     90*UO**2*DHO*DDCO/CO**2-6.0*UO**2*DHO*DCO**2/CO**3+3.0*RO*DDDRO*DHO
     1*DCO/CO**2+3.0*RO*DDRO*DHO*DDCO/CO**2-6.0*RO*DDRO*DHO*DCO**2/CO**3
     2+3.0*RO*UO*DDHO*DDCO/CO**2-6.0*RO*UO*DDHO*DCO**2/CO**3+RO*UO*DHO*D
     3DDCO/CO**2-6.0*RO*UO*DHO*DCO*DDCO/CO**3+6.0*RO*UO*DHO*DCO**3/CO**4
     4-RO*DDDDRO*DHO/CO)/(RO*(CO-UO))
      T=0.0
      DO 10 I=1,4
      YY(I,1)=RO+UO*T+DDRO*T**2/2.0+T*DDDRO*T**2/6.0+T**2*DDDDRO*T**2/24
     1.0+T**3*DDDDUO*T**2/120.0
      YY(I,2)=UO+DDRO*T+DDDRO*T**2/2.0+T*DDDDRO*T**2/6.0+T**2*DDDDUO*T**
     12/24.0
      YY(I,4)=PO*(RO/YY(I,1))**(3.0*G)
      CALL ACCEL(YY(I,1),YY(I,2),YY(I,3),YY(I,4))
      XX(I)=T
   10 T=T+S
      R1=YY(1,1)
      R2=YY(2,1)
      R3=YY(3,1)
      R4=YY(4,1)
      U1=YY(1,2)
      U2=YY(2,2)
      U3=YY(3,2)
      U4=YY(4,2)
      DU1=YY(1,3)
      DU2=YY(2,3)
      DU3=YY(3,3)
      DU4=YY(4,3)
      UP4=U4
      RP4=R4
      C4=U4
      D4=R4
      I=4
  715 I=I+1
  250 L=0
  700 UP5=U1+4.0*S*(2.0*DU4-DU3+2.0*DU2)/3.0
      RP5=R1+4.0*S*(2.0*U4-U3+2.0*U2)/3.0
  230 COD=UP5-112.0*(UP4-C4)/121.0
      DOD=RP5-112.0*(RP4-D4)/121.0
      POD=PO*(RO/DOD)**(3.0*G)
      CALL ACCEL(DOD,COD,DCOD,POD)
      C5=(9.0*U4-U2+3.0*S*(DCOD+2.0*DU4-DU3))/8.0
      D5=(9.0*R4-R2+3.0*S*(COD+2.0*U4-U3))/8.0
      IF(ABS(UP5-C5).LT.1.0)GO TO 210
  260 UP5=C5+9.0*(UP5-C5)/121.0
      RP5=D5+9.0*(RP5-D5)/121.0
      L=L+1
      IF(L.LT.2)GO TO 230
C   HALF INTERVAL PROCEDURE
```

```
  400 S=S/2.0
      R4MH=(80.0*R4+135.0*R3+40.0*R2+R1)/256.0+S*(-15.0*U4+90.0*U3+15.0*
     1U2)/128.0
      R3MH=(12.0*R4+135.0*R3+108.0*R2+R1)/256.0+S*(-3.0*U4-54.0*U3+27.0*
     1U2)/128.0
      U4MH=(80.0*U4+135.0*U3+40.0*U2+U1)/256.0+S*(-15.0*DU4+90.0*DU3+15.
     10*DU2)/128.0
      U3MH=(12.0*U4+135.0*U3+108.0*U2+U1)/256.0+S*(-3.0*DU4-54.0*DU3+27.
     10*DU2)/128.0
      R1=R3MH
      R2=R3
      R3=R4MH
      U1=U3MH
      U2=U3
      U3=U4MH
      PH=PO*(RO/R1)**(3.0*G)
      CALL ACCEL(R1,U1,DU1,PH)
      DU2=DU3
      PH=PO*(RO/R3)**(3.0*G)
      CALL ACCEL(R3,U3,DU3,PH)
      L=L+1
      IF(L.GT.10)WRITE(6,75)
   75 FORMAT(58H THE PROCESS IS NOT CONVERGING QUICKLY ENOUGH AT SOME ST
     1EP)
      GO TO 700
C  CALCULATION OF FINAL VALUES
  210 YY(I,2)=C5+9.0*(UP5-C5)/121.0
  500 YY(I,1)=D5+9.0*(RP5-D5)/121.0
      YY(I,4)=PO*(RO/YY(I,1))**(3.0*G)
      CALL ACCEL(YY(I,1),YY(I,2),YY(I,3),YY(I,4))
      XX(I)=XX(I-1)+S
      IF(I.GT.999)GO TO 7
C  RELOCATE CERTAIN QUANTITIES FOR THE NEXT STEP OF THE INTEGRATION
      C4=C5
      D4=D5
      UP4=UP5
      RP4=RP5
      R1=R2
      R2=R3
      R3=R4
      R4=YY(I,1)
      U1=U2
      U2=U3
      U3=U4
      U4=YY(I,2)
      DU1=DU2
      DU2=DU3
      DU3=DU4
      DU4=YY(I,3)
      GO TO 715
    7 DO 9 J=1,I
      Y=YY(J,1)*YY(J,2)**2/2.0+YY(J,1)*(YY(J,4)-PL)/DL*(1.0-(YY(J,4)-PL)
     1/(2.0*DL*CL**2))
      XK=(CL**2/Y**2)*CL*YY(J,1)**2*YY(J,2)*(1.0-YY(J,2)**2/(2.0*CL**2))
     1-CL**2*YY(J,1)/Y*(1.0-YY(J,2)/CL)
      U(J)=Y/(CL*STF)+XK/(CL*STF**2)*(Y/CL)**2*(1.0-Y/(CL**2*STF)+(XK/CL
     1)**2*(Y/CL)**4/(2.0*STF**4*CL**2))
      UU(J)=Y/(CL*STF2)+XK/(CL*STF2**2)*(Y/CL)**2*(1.-Y/(CL**2*STF2)
     1 +(XK/CL)**2*(Y/CL)**4/(2.*STF2**4*CL**2))
      U=U(J)
```

```
      YY(J,5)=DL*(Y/STF-U**2/2.0)+DL/(2.0*CL**2)*(Y/STF-U**2/2.0)**2
      U=UU(J)
      YYY(J)=DL*(Y/STF2-U**2/2.)+DL/(2.*CL**2)*(Y/STF2-U**2/2.)**2
      TAU(J)=XX(J)+(STF-YY(J,1))/CL*(1.0-YY(J,2)*YY(J,1)/(CL*STF))
      TAU2(J)=XX(J)+(STF2-YY(J,1))/CL*(1.-YY(J,2)*YY(J,1)/(CL*STF2))
      TAU(J)=TAU(J)*1.0E3
      IF(TAU(J).LT.TAU(J-1))   III=J
      TAU2(J)=TAU2(J)*1000.
    9 XX(J)=XX(J)*1.0E3
      WRITE(6,16)H,RO,PO,G,B,EN,DLREF,STF,STF2
   16 FORMAT(1H1//////////10X,42HGILMORES SECOND ORDER APPROXIMATION FOR
     1A ,F6.0,12H FOOT WATER  //11X,21HDEPTH IMPLOSION OF A ,F5.1,32H IN
     2CH RADIUS SPHERE FILLED TO A //9X,12HPRESSURE OF ,F6.2,37H PSI WIT
     3H A GAS WHOSE GAMMA VALUE IS ,F6.3//9X,36HB, N, AND DENSITY OF THE
     4 LIQUID ARE ,F8.0,5H PSI ,F6.3,5H AND //9X,E10.4,32H LBS-SEC**2/IN
     5**4 RESPECTIVELY. ,14HSTANDOFFS ARE ,F6.2,8H IN AND ,F6.2,3H IN)
        IF(III.GT.0)    WRITE(6,18)TA       U(III)
   18 FORMAT(1H /////////10X,72H NON-UNIQUE-VALUES APPEAR FOR STANDOFF TI
     1MES IN REGION PRECEIDING TSTF1=,F8.5,10H MILLISECS  )
      WRITE(6,11)(XX(J),YY(J,1),YY(J,2),YY(J,3),YY(J,4),TAU(J),YY(J,5),
     1U(J),TAU2(J),YYY(J),UU(J),J=1,I)
   11 FORMAT(123H1  TIME      RADIUS    WALL VEL         ACCEL        WALL PRES
     1        T STF1        P STF1        U STF1        T STF2       P STF2        U ST
     2F2      /35H VALUES IN MILLISECS, IPS, AND PSI.
     9        /////(1X,F9.6,F8.5,E13.5,E13.5,E12.5,F11.6,E12.4,E12.4,
     3 F10.6,E12.4,E12.4/))
C  IF THIS PROGRAM IS BEING REPRODUCED AND USED ON A COMPUTER FACILITY
C  OTHER THAN THAT AT N.S.R.D.C. ELIMINATE ALL CARDS BETWEEN THIS CARD
C  AND THE NEXT COMMENT CARD WHICH READS - END OF PLOTTING ROUTINE -
      DATA SCALE/6HLINEAR/
      XL=0.0
      CALL SPACE(XX(I),XR,DX)
      DX=2.0*DX
      Y2MAX=YY(1,2)
      Y2MIN=YY(1,2)
      Y4MAX=YY(1,4)
      Y5MAX=YY(1,5)
      Y5MIN=YY(1,5)
      UMAX=U(1)
      UMIN=U(1)
      DO 90 N=1,I
      YY(N,3)=U(N)
      IF(YY(N,2).LT.Y2MAX)GO TO 81
      Y2MAX=YY(N,2)
   81 IF(YY(N,2).GT.Y2MIN)GO TO 82
      Y2MIN=YY(N,2)
   82 IF(YY(N,4).LT.Y4MAX)GO TO 83
      Y4MAX=YY(N,4)
   83 IF(YY(N,5).LT.Y5MAX)GO TO 84
      Y5MAX=YY(N,5)
   84 IF(YY(N,5).GT.Y5MIN)GOTO850
      Y5MIN=YY(N,5)
  850 IF(YYY(N).LT.Y5MAX)GOTO855
      Y5MAX=YYY(N)
  855 IF(YYY(N).GT.Y5MIN)GOTO85
      Y5MIN=YYY(N)
   85 IF(U(N).LT.UMAX)GO TO 86
      UMAX=U(N)
   86 IF(U(N).GT.UMIN)GO TO 87
      UMIN=U(N)
```

```
87 IF(UU(N).LT.UMAX) GOTO 88
   UMAX=UU(N)
88 IF(UU(N).GT.UMIN) GOTO 80
   UMIN=UU(N)
80 CONTINUE
   CALL SPACE(RO,YT,DY)
   READ(5,76)BTITLE(1),BTITLE(2),BTITLE(3),BTITLE(4),BTITLE(5),BTITLE
  1(6),BTITLE(7),BTITLE(8)
76 FORMAT(10A6)
   DATA(TITL(N),N=1,5)/6HTIME I,6HN MSEC,3*6H      /
   DO 70N=1,5
70 TITLEX(N)=TITL(N)
   DATA(TITLE(J),J=1,25)/6HRADIUS,6HIN IN ,3*6H      ,6HVELOCI,
  16HTY IN ,6HINCHES,6H PER S,6HECOND ,6HWALL P,6HRESSUR,6HE IN P,
  26HSI    ,6H      ,6HEULERI,6HAN VEL,6HOCITY ,6HIN IPS,6H      ,
  36HOVERPR,6HESSURE,6H IN PS,6HI     ,6H      /
   DO71  MI=1,5
71 TITLEV(MI)=TITLE(MI)
   CALL GPLOT(SCALE,1,I,0,XL,XR,0.0,YT,DX,DY)
   CALL SPACE(Y2MAX,YT,DYT)
   Y2MIN=-Y2MIN
   CALL SPACE(Y2MIN,YB,DYB)
   YB=-YB
   DY=AMAX1(DYB,DYT)
   DO72  MI=1,5
   II=MI+5
72 TITLEV(MI)=TITLE(II)
   CALL GPLOT(SCALE,2,I,0,XL,XR,YB,YT,DX,DY)
   CALL SPACE(Y4MAX,YT,DY)
   DO73  MI=1,5
   II=MI+10
73 TITLEV(MI)=TITLE(II)
   CALL GPLOT(SCALE,4,I,0,XL,XR,0.0,YT,DX,DY)
   CALL SPACE(UMAX,YT,DYT)
   UMIN=-UMIN
   CALL SPACE(UMIN,YB,DYB)
   YB=-YB
   DY=AMAX1(DYB,DYT)
   DO 55 N=1,I
   YY(N,3)=U(N)
55 XX(N)=TAU(N)
   A=AMAX1(TAU(I),TAU2(I))
   CALL SPACE(A,XR,DX)
   DX=2.0*DX
   DO74  MI=1,5
   II=MI+15
74 TITLEV(MI)=TITLE(II)
   CALL GPLOT(SCALE,3,I,1,XL,XR,YB,YT,DX,DY)
   DO155 N=1,I
   YY(N,3)=UU(N)
155 XX(N)=TAU2(N)
   CALL GPLOT(SCALE,3,I,2,XL,XR,YB,YT,DX,DY)
   CALL SPACE(Y5MAX,YT,DYT)
   Y5MIN=PL
   CALL SPACE(Y5MIN,YB,DYB)
   YB=-YB
   DY=AMAX1(DYB,DYT)
   DO156N=1,I
156 XX(N)=TAU(N)
   DO77  MI=1,5
```

```
      II=MI+20
   77 TITLEV(MI)=TITLE(II)
      CALL GPLOT(SCALE,5,I,1,XL,XR,YB,YT,DX,DY)
      DO56N=1,I
      YY(N,5)=YYY(N)
   56 XX(N)=TAU2(N)
      CALL GPLOT(SCALE,5,I,2,XL,XR,YB,YT,DX,DY)
C  END OF PLOTTING ROUTINE
      GO TO 22
  999 STOP
      END
$IBFTC ACCELS
      SUBROUTINE ACCEL(R,U,DU,P)
      COMMON/AC/CL,PL,DL,B,EN,G,PO,RO
      C=CL*((P+B)/(PL+B))**((EN-1.0)/(2.0*EN))
      H=EN*(PL+B)/((EN-1.0)*DL)*(((P+B)/(PL+B))**((EN-1.0)/EN)-1.0)
      DP=-3.0*G*PO*U/R*(RO/R)**(3.0*G)
      DH=DP/DL*((PL+B)/(P+B))**(1.0/EN)
      DU=U**2/(2.0*R)*(U-3.0*C)/(C-U)+H*(C+U)/(R*(C-U))+DH/C
      RETURN
      END
$IBFTC SPACES  NODECK,SDD
      SUBROUTINE SPACE(ENDPT,XR,DX)
      IF(ENDPT.LE.1.0E-3)GO TO 68
      XID=ALOG10(ENDPT*1000.0)-3.0
      IF(XID.GE.0.0)GO TO 114
      ID=XID-1.0
      GO TO 61
  114 ID=XID
      IF(ID.GE.1)GO TO 366
   61 IXR=ENDPT*10.0**(1-ID)
      XR=IXR+1
      XR=XR/10.0**(1-ID)
      IF(IXR.LE.20)GO TO 361
      DX=IXR/20
      DX=DX/10.0**(1-ID)
      GO TO 111
  361 DX=1.0/10.0**(1-ID)
      GO TO 111
  366 IXR=ENDPT/10.0**(ID-1)
      XR=IXR+1
      XR=XR*10.0**(ID-1)
      IF(IXR.LE.20)GO TO 136
      DX=IXR/20
      DX=DX*10.0**(ID-1)
      GO TO 111
  136 DX=1.0*10.0**(ID-1)
      GO TO 111
   68 WRITE(6,67)
   67 FORMAT(1H1///10X,46H THE ENDPOINT IS TOO SMALL FOR THIS SUBROUTINE
     1)
  111 RETURN
      END
$IBLDR HOLLV                                                    HOLL0000
$IBLDR PLOTV                                                    PLOT0000
$IBLDR VCHARV                                                   VCHA0000
$IBLDR TABL1Q                                          10/10/65 TABL0000
$IBLDR RITE2Q                                                   RITE0000
$IBLDR APRNTV                                           6/15/65 APRN0000
$IBLDR APLOTV                                          10/04/65 APLO0000
```

For each bubble collapse, information must be read in on data cards in the following way:

Card 1

Cols 1-12	collapse depth in feet of water,	F12.5
Cols 13-24	initial sphere radius in inches,	F12.5
Cols 25-36	initial internal gas pressure in psi	F12.5
Cols 37-48	the specific heat ratio γ for the gas inside the sphere,	F12.5
Cols 49-60	#1 standoff in inches,	F12.5
Cols 61-72	#2 standoff in inches	F12.5
Cols 73-80	the value of n, the exponent in the equation of state for the adiabatic compression of the liquid	F 8.5

Card 2

Cols 1-15	the value of B in psi, a constant in the equation of state for the adiabatic compression of the liquid,	E15.5
Cols 16-30	the density of the liquid in lb-sec^2/in^4 at standard temperature and pressure,	E15.5
Cols 31-45	the sound speed in the liquid in in/sec at standard temperature and pressure.	E15.5

Card 3 contains graph labels. If the program is not being used at NSRDC, then this card must not be included in the data.

Card 3

Cols 1-48 Main graph title for all graphs.

Two blank cards in succession stop the computer. Each case requires no more than 3 minutes running time.

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13. ABSTRACT

Two methods are presented for calculating the instantaneous pressure, velocity, acceleration, and radius associated with the collapse of a spherical gas-filled cavity in an infinite compressible liquid. One is based on the ideal gas law, the other is based on the Beattie-Bridgeman equation of state for the gas inside the cavity. In most cases the latter assumption must be restricted to relatively mild implosions. The good agreement between the two methods serves to verify their validity.

Included are listings of the two Fortran IV computer programs used to obtain numerical results of the analyses based on the ideal and Beattie-Bridgeman gas models. The influence of several different gases, initial internal gas pressures, and liquids on the collapse is studied. On the basis of explanations of the resulting behavior, new methods of producing similar behavior are discussed.

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Bubble Implosion						
Gas Bubble Implosion						
Buoyancy Sphere						
Beattie-Bridgeman Equation of State						

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(PAGE 2)

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